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ADIANT ENERGY
AND THE
PHTHALMIC LENS

BOOTH

RADIANT ENERGY AND THE OPHTHALMIC LENS

BOOTH

RADIANT ENERGY AND THE OPHTHALMIC LENS

BY

FREDERICK BOOTH

INTRODUCTION BY

WHITEFIELD BOWERS, A.B., M.D.

FORMERLY MAJOR, M. C., U. S. A.

230 ILLUSTRATIONS

PHILADELPHIA
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In Celebration of my Mother's
Seventy-third Birthday
This Volume
Is Affectionately Dedicated

PREFACE

The object of my effort is to present from a didactic standpoint a study of the principles of optics. Also the eye from an optical standpoint—accommodation, convergence, the ophthalmic lens, transposition, etc. Higher mathematics have been omitted as far as possible.

In order to obtain a better understanding of optics, I have included radiant energy. These sections describe the ether theory. Also light, actinic, heat, electrical, cathode, X-ray, alpha, beta, and gamma rays; sound and water waves; also waves in a rope. Harmonic motion is described. The various theories of light are explained.

The drawings have been made self-explanatory as far as possible.

A writer's skill lies in his ability to accomplish two things: First, the selection of proper material; second, the plain and direct presentation of the subject. Foreign matter such as superfluous words, useless explanation, obsolete methods and illustrative cases being excluded.

Condensing a manuscript, by this is meant to properly draw the line of demarkation between paucity and superfluity, involves talent of a high order combined with an enormous volume of work. In view of the above stated facts it is many times easier to write a treatise of 100,000 words than one of 50,000 words. It therefore follows that the value of a book cannot be determined by its bulk or size.

With the exception of a few dates and names of investigators mentioned in the chapter on radiation where it is a self-evident essential, I have carefully excluded the historical.

I do not claim that the bibliographical, historical, obsolete methods and illustrative cases do not have a place in optical literature. On the contrary a volume of such would be of much value to an advanced student as it would certainly broaden his views.

The student of science will note that the effect of an ether wave is determined by its wave length and rate of vibration. We have the electrical heat, light, actinic and X-ray as examples. Waves within a certain range have been used in curing various diseases where drugs have failed. If allowed to theorize I might add that lying hidden in the invisible and possibly visible portions of the spectrum are forces that, when properly harnessed, will revolutionize medical science; also replace steam and oil as propulsive agents. I would love to devote years of research work along these lines but the opportunity is not available.

Whitefield Bowers, A.B., M.D., sacrificed much valuable time in collaboration with the author in outlining this work and constructing the text. This occurred many years ago. When the work was finally completed, I looked for a properly qualified and sound critic and Dr. Bowers, again volunteered many valuable hours to a very careful reading of the manuscript before expressing his opinion. Such exemplary unselfishness is deeply appreciated by the author.

I drew heavily upon the text books of the LaPorte Public Library. The pleasant influence of this really efficient institution did much to lessen the drudgery involved in book writing.

F. R. Warren, M.D. of Michigan City, Ind., placed at my disposal his library which proved to be of much value, and for which I am very grateful.

The F. A. Hardy Co., Chicago, Ill., have been kind enough to lend some of their pictures. Also descriptive material for the Ophthalmometer and their instructions for taking spectacle measurements, for which I am truly thankful.

Dr. Bowers, has fittingly stated that book writing is a mathematical proposition, meaning that the perfect book is a possibility. This is my goal and any suggestions by readers that will improve future editions will be deeply appreciated.

F. B.

LA PORTE, IND.
SOUTH BEND, IND.
MICHIGAN CITY, IND.

INTRODUCTION

There have been many books written along the lines covered by this one, and each has some special claim to value. But where the entire field has been covered, only the "high spots" have been adequately touched, and where special care has been given to certain lines, much of the remainder has not received any attention.

Mr. Booth, in collaboration with others, has worked long on this subject and published a booklet which embodied their labors. But to his thoroughgoing mind the subject had not been completely worked out and put together to his satisfaction. Therefore after a number of years, he mapped out the subject again, aided by his larger experience and extensive literature on the subject. Then painstakingly and with love for the subject he gave himself to clearing the field and like a landscape gardner bringing out the features of the terrain.

I believe that Mr. Booth, has gathered more facts, from all available sources; worked out more lines of reasoning, and secured a more comprehensive entity than has been heretofore accomplished along these lines. He has left nothing to the imagination, therefore a novice can begin with him and, each step being worked out so well, can follow him to a thorough understanding. For the man who has studied along these lines as a part of his armamentarium this book is a most concise and comprehensive compendium for reference. It affords a grounding in fundamentals that is needed by the many who have not had access to the scattered bits, nor the time—nor the inclination perhaps,—to work it out for themselves.

WHITEFIELD BOWERS, A.B., M.D.
Formerly Major, M.C., U. S. A.

MICHIGAN CITY, INDIANA.

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RADIANT ENERGY AND THE OPHTHALMIC LENS

CHAPTER I

ANATOMY

The anatomy of the eye which is of interest to the student, is illustrated in the horizontal section Fig. 1.

1. **Cornea.**—Forms the anterior end of the globe and about one-sixth of the external tunica, Approx. ellipsoidal shape. Flatter in horizontal meridian. Index of refraction 1.342.

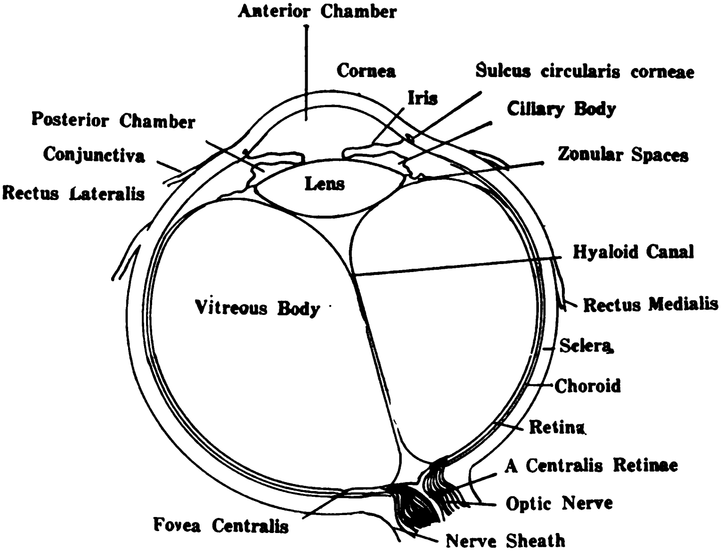


FIG. 1.

Refractive Power of Anterior Surface + 47.24D. Posterior surface −4.73. Five layers as follows: (1) Conjunctival epithelium. (2) Ant. elastic lamina. (3) Substantia propria or true corneal layer. (4) Post. elastic lamina, or Descemet's membrane, (5) Post. epithelium.

2. **Crystalline Lens.**—A bi-convex transparent elastic body. Enclosed in its capsule and suspended by a ligament. Posterior surface more convex than anterior surface. Curvature flatter in horizontal meridian. Index of refraction 1.44 to 1.45. Refracting power varies.

Anterior surface plus 6.13D. Posterior surface plus 9.53D. (Fig. 2.)

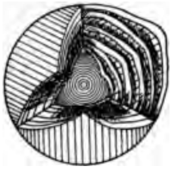


FIG. 2.

The lens has a nucleus over which the layers fold.

3. **Aqueous Humor.**—A transparent serous fluid, slightly viscid. Fills both anterior and posterior chambers. Index 1.336.

4. **Vitreous Humor.**—Occupies the cavity back of lens. A transparent gelatinous substance. Index of refraction 1.336.

5. **Muscles.**—There are six muscles that produce movement of the eyeballs. A simple movement is produced by one muscle. Where a compound movement occurs two or more muscles act in combination. (Figs. 3, 4.)

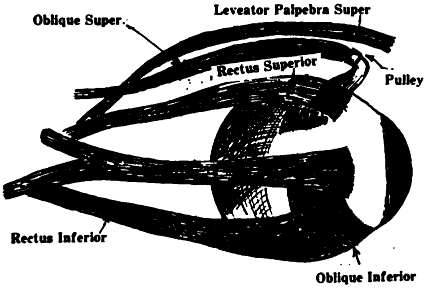


FIG. 3.

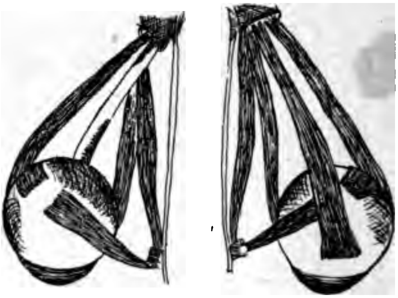


FIG. 4.

Inward, Internal Rectus. Outward, External Rectus. Downward, Inferior Rectus and Superior Oblique. Upward, Superior Rectus, and Inferior Oblique. Upward and inward, Superior and Internal Recti and Inferior Oblique. Upward and outward, Superior and External Recti and Inferior Oblique. Downward and inward, Inferior and Internal Recti and Superior Oblique. Downward and outward, Inferior and External Recti and Superior Oblique.

6. The Retina.—The retina is a delicate membrane found between the vitreous and choroid. Upon its surface the retinal image is formed. It is made up of ten layers:

1. Membrana limitans interna.
2. Optic nerve fibers.
3. Ganglion Cells.
4. Internal molecular.
5. Internal granules.
6. External molecular, or inter-granular.
7. External granules.
8. Limitans Externa.
9. Rods and cones or Jacob's membrane.
10. Pigment layer.

7. Yellow Spots or Macula Lutea.—The macula is that part of the retina where most acute vision is obtained. Located about .1″ to the temporal side of the center of the optic disc. At the center of the macula is located a depression, the fovea centralis. This point is the area of most acute vision.

For a complete chapter the reader is referred to the Morris' Anatomy.

CHAPTER II

RADIANT ENERGY

8. The Ether Theory.—In the study of radiant energy we assume that not only interstellar space, but all space, even that between the molecules and atoms of matter is pervaded by a substance through which the various forms of energy may be conveyed.

Long ago this hypothesis was advanced and the substance defined as the Ether. The ether is described as an extremely thin substance of enormous elasticity. Maxwell has calculated its rigidity as follows: 10^{-9} that of steel and its density 936×10^{-21} that of water at 4°C., or equal to that of our atmosphere at a height of 210 miles—a density vastly greater than that of the same atmosphere in the interstellar spaces. Large bodies, such as the planets pass freely through it and the ether passes through the planets and suffers but slight retardation, if any. This being true the substance in question must be an almost perfect fluid; of immeasurable elasticity and practically incompressible. Lord Kelvin has pointed out that in structure it is much finer grained than water, glass and metals. The existence of this substance cannot be proven by any known means of analysis. It cannot be weighed, measured or compressed sufficiently to be recorded.

The only available proof of its existence lies in our ability to account for many physical phenomena, *e.g.*, radiant energy. It might be stated here that some modern writers reject the ether theory on the grounds that we cannot account for all radiant phenomena. One thing seems to have been well established and that is the Huyghens transverse wave theory, the discussion of which will be taken up in due order.

9. Radiation.—Energy which is transmitted though space is from one body to another is defined as radiant energy. When the molecules of a substance are agitated (we might illustrate this by striking a match) periodic transverse waves are released, which traverse space at a measurable speed. If we still cling to the ether theory it is not because we are able to explain the exact nature of these ether disturbances. Whether these waves are due to changed position of the ether particles or a condition of strain, and release, we cannot state. Many demonstrations have been so conclusive as to establish the periodic transverse wave theory. In the study of radiant energy a survey of the different waves viz.: water waves, sound waves and radiant energy, will be of interest and profit. In order to grasp the full meaning of the various waves we must first study motion of translation and rotation.

10. Translation and Rotation.—Pure translation is indicated when every point of a body moves with the same velocity and no plane in the body suffers a change in direction. The motion may be up and down.

The body may have a motion of *pure rotation* when every point in the body describes a circle around the axis of rotation. For an illustration we might take a circular disc with a radius of 12 inches. From the center we will strike a circle with a 6-inch radius. It is self evident that a point on the 12-inch and 6-inch circles make a complete revolution in the same length of time. As the circumferences of circles are proportional to their radii the velocity of a point varies directly as its distance from the axis. From this we obtain a velocity on the 12-inch radius to be double that of the 6-inch radius.

We have a common demonstration of a combined translation and rotation motion in a wheel rolling over the ground.

11. Harmonic Motion.—In Fig. 5, B, represents a body suspended from a string and swung in a horizontal plane. If an eye is placed directly over or under the point where the string is fixed, this path will appear circular. But if an eye is placed in the same plane as that occupied by the body it will appear to travel to and fro in this plane. It will be observed that as the body passes over the center its velocity will gradually diminish until the end of the path is reached.

During its return trip the velocity will increase, until the center is reached. This phenomena may be described as ***uniform circular motion*** projected up on a diameter.

We will let Fig. 6, represent a body traveling in a circle at a uniform speed. As the body moves to *a*, it appears to be at 1. As it moves to the next period *b*, its apparent position is at 2, and the velocity increasing. This occurs until the body has passed through the points *c*, and *C*. The velocity then appears to diminish as the succeeding points *d*, *e*, *f*, 4.5.6. and *D*, are reached, when a period of apparent rest is observed. The points *D.D*–1 represent the apparent rectilinear path. If we divide the circle into an equal number of parts say 12, the body will cover each arc in ⅙ of a second. ***Harmonic*** motion occurs when the body traverses the points on line ***D.D*–1** in equal periods of time. The vibrations of the tuning fork, the points of a vibrating string; light waves and electrical waves in the ether are examples of harmonic motion.

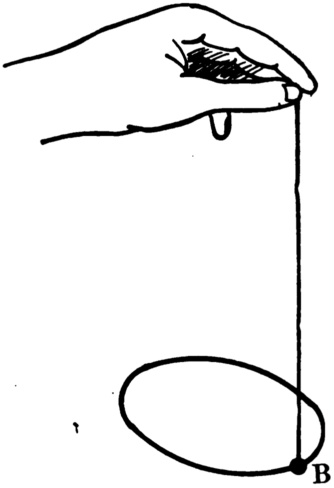


FIG. 5.

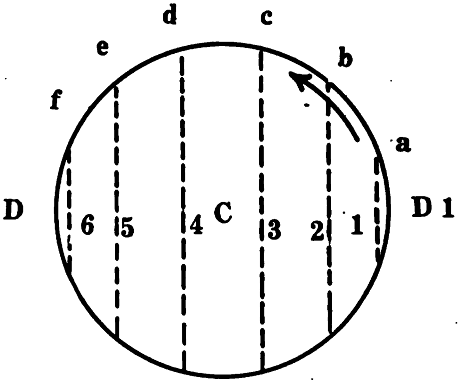


FIG. 6.

This motion is *Periodic* as it repeats itself at regular intervals. The *Period* is represented by the time required for the object to travel from *D*, to *D*–1, and back again. The period is then the time required for the body to travel around the circle. The motion is *Positive* if the apparent movement is from left to right. If from right to left, *Negative*. The distance *C*, *D*, or *C*, *D*–1 is the *amplitude* of motion. The distance for such as *C*, 5, is the displacement. *D*, and *D*–1 are referred to as the points of greatest elongation. By *Phase* is meant the fraction of a period since the particle last passed through *C*, in a positive direction. When the body returns from *D*, to *D*–1 periodically but at a slow rate, such as a pendulum it is said to oscillate. If the motion is very rapid it is referred to as *vibration*.

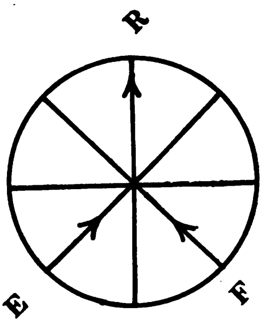


Fig. 7.

To compound harmonic motion is quite difficult in some cases. Suppose that we have several motions of the same period and traveling in the same direction. The resultant will be a period unchanged but the amplitude will equal that of the several component forces. If a body is acted upon by two harmonic motions of the same direction but of different phase by half a period, the resultant will have an amplitude equal to the difference of amplitudes of the component forces. If we have two forces acting at right angles, whether of the same phase or differing by a ½ period, we may find the resultant as follows: Let the two forces *F*–*E*, be represented by two lines at right angles, Fig. 7, the resultant *R*, will be represented by a line diagonal to *F*, and *E*.

The process of finding the *resultant* becomes quite complex, if the phase is ¼ period as the direction of the resultant becomes a circle or ellipse. For a further discourse the reader is referred to works on mathematics where the subject covers many pages.

12. Harmonic Curve.—Fig. 8. We will assume that the harmonic motion of a body is on the line *l*–*la*, and traveling from left to right. The circle is divided into fourteen parts of equal length, 1, 2, 3, 4, 5, 6 and 7. Draw lines through these points cutting line l, l–a. In this way the points *C*, *D*, *E*, *F*, *G* and *H*, are found, which locate the body relative to the harmonic motion at equal intervals of 1/14 of the period. Lines parallel with *l*, *l–a*, divide the space into equal intervals. Each space locates the body during 1/14 of a period. If we take the combinations *l*, *l–a*, with *l*, *C*, and 2–3 with *C*, –*D*, etc. the points 1, 2, 3, 4, etc. are found which locates the body after intervals of 1/14 period. If the points are joined we have an approximation of the *Harmonic Curve*. This curve represents the *Resultant* of harmonic motion a uniform rectilinear motion at right angles.

13. **Wave Motion.**—Fig. 8 starting at 1, the body travels along the curve 1, 2, 3, etc. Each interval represents a 1/14 period. The body will be found in the points 1, 2, 3, 4, etc. after a time equal to two periods. This illustration conveys a good idea as to how waves are propagated in an elastic media.

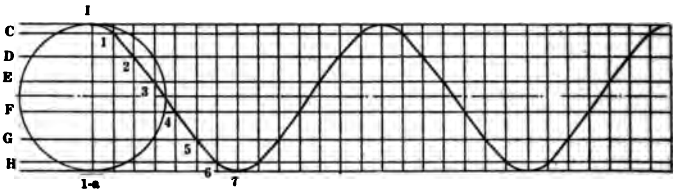


FIG. 8.

14. **A transverse wave** is one in which the harmonic motion is at right angles to the direction in which it is traveling.

15. **A longitudinal wave** is one in which the vibration of the particles are parallel with the direction in which the waves are traveling.

When these two are combined the particles move in ellipses.

16. **Waves in a Rope and Water.**—Fig. 9. If a long rope is fixed at one end and the other end shaken up and down a wave will be started at the hand and move toward the fixed end. It will be observed that the particles vibrate to and fro across the line in which the wave travels, but the particles do not move forward with the wave. If a pool of water is disturbed waves are created but an

object resting upon the surface is not carried along, it simply moves up and down.

The crest is represented by the distance from 1 to 5. The *trough* is the distance 3 to 7. A *wave length* is the distance A to 4, —2 to 6 etc. The *vibration period* is the time required for the wave to advance one *wave length.*

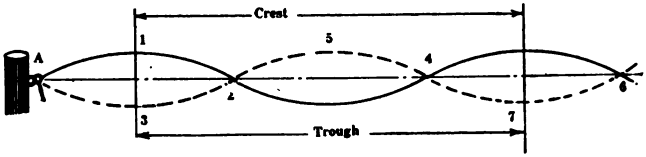


Fig. 9.

In water waves the particles move in eclipses. Water waves are a combination of transverse and longitudinal waves.

17. Sound Waves.—Sound waves are described as longitudinal waves of *compression and rarefaction.*

Fig. 10. When the bell vibrates, the air is alternately compressed *A*, and rarefied *B*. When the area *A*, is compressed the molecules act upon those of the next layer, putting them in a state of agitation and causing them to become compressed. The molecules of *A* rebound and *A*, then passes into a state of rarefaction. The air does not move forward with the wave; the molecules simply move to and fro a very small distance. When these waves impinge upon an ear the sensation of hearing is produced.

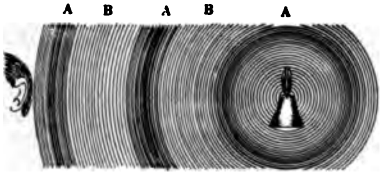


Fig. 10.

The wave length and velocity may be obtained as follows: l = wave length. n = number of vibrations per second of the vibrating body. Wave length is equal to velocity divided by the number of vibrations per second. $l = \frac{v}{n}$. And the velocity is equal to the number of vibrations per second times the wave length—$v = nl$.

Continuous sound waves are produced by the vibrating fork Fig. 11. Each wave in air will travel approximately 1100 feet per second. If the fork vibrates 110 times per second the pulses will be 10 feet apart. *The distance between adjacent pulses represents the wave length.*

18. Velocity.—The velocity at which sound waves travel depends upon the elasticity and density of the medium.

It varies directly proportional to the square root of the elasticity, and inversely proportional to the square root of the density.

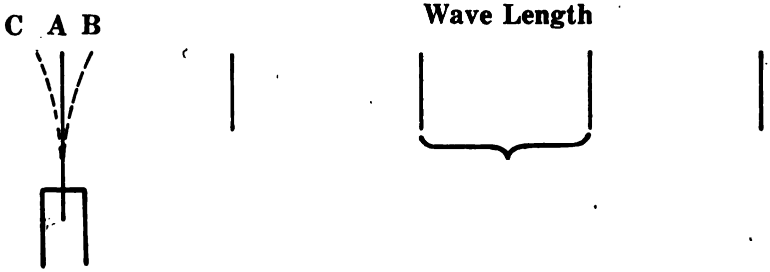


FIG. 11.

Newton produced a formula for the velocity of sound which was corrected by Laplace as follows:

$$V = \sqrt{1.41 \frac{P}{D}}$$

V = the velocity in centimeters per second.

P = the pressure of the gas per qcm. in dynes.

D = the density of the gas in grams per ccm.

1.41 is the ratio of the specified heat of gas under constant pressure to its specific heat, under constant volume.

For air $P = 1033 \times 981$. $D = .001293$. According to the formula $V = 33{,}240$ cm. approx.

Liquids and solids are possessed of great elasticity as compared with their density. Therefore they transmit sound with a high velocity. In air at the freezing temperature (0°C., or 32°F.) approx. 332 m. or 1,090 feet per second. Oxygen is 16 times denser than hydrogen. If the pressure of the two media are the same, the elasticity is identical. Sound travels four times as fast in hydrogen as it does in oxygen. If the pressure is changed the velocity of sound is not altered because pressure effects density and elasticity equally. In water at 8°C. sound travels at the rate of 4,708 feet per second. Glass at 14,850 feet per second. Iron at 16,820 feet per second. Lead possessed of great density but slight elasticity transmits sound waves at but 4,030 feet per second.

19. Reflection.—Sound waves are reflected according to the laws of reflected motion.

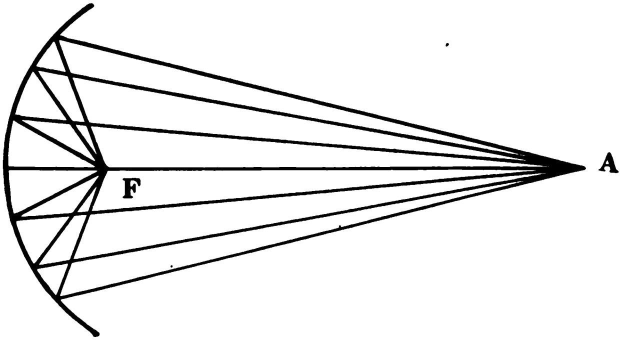


Fig. 12.

When the bodies are perfectly elastic, the angle of incidence is equal to the angle of reflection, and lie in the same plane. When the elasticity of the bodies is imperfect the angle of reflection is greater than the angle of incidence.

Sound waves can be brought to a focus by a concave mirror as in the case of light waves. This can be easily demonstrated by placing a watch at such a distance *A*, Fig. 12, as to be scarcely audible. Then if the ear is placed at about the principal focus *F*, of the mirror the ticking noise will be greatly magnified.

An echo is an example of reflected sound.

If two mirrors are arranged as in Fig. 13 and a watch suspended at *A*, the waves will be reflected to the point *B*, and although the space is as much as three hundred feet, under favorable conditions an ear in the vicinity of *B*, can hear the ticking. *A* and *B* are conjugate.

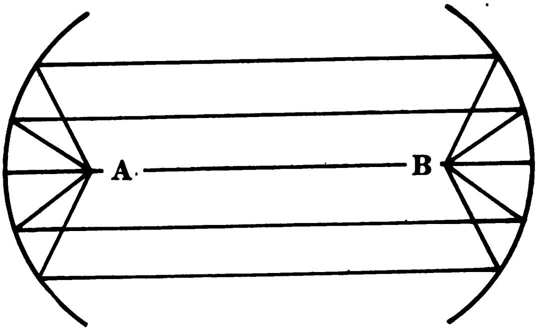


Fig. 13.

20. Refraction.—Sound waves may be refracted.

In Fig. 14 the lens has flexible walls. A rubber toy balloon charged with carbon dioxide is suitable. Waves starting at *A* will be refracted to a focus at *B*.

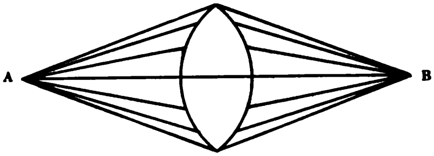


Fig. 14.

21. Interference.—If two simultaneous waves which agree in period and amplitude but are of opposite phase that is, the difference is half a wave length, the waves will destroy each other and silence result, Fig. 31.

Ether Dynamics

22. Newton's Emission Theory of Light.—Newton advanced the theory that luminous bodies throw off extremely minute elastic particles which traverse transparent bodies at an enormous speed, and also excite the sensation of light by impact upon the retina. This theory accounts for the laws of distance, reflection and refraction. There are certain optical phenomena that cannot be reasonably explained from Newton's hypothesis.

23. **Einstein's Gravitational Theory on Light.**—Dr. Albert Einstein, propounded his theory of relativity almost fifteen years ago. He claimed that light responds to gravitation and could be demonstrated during a total eclipse of the sun. The fixed stars selected for observation purposes were those close to the sun's disc. Observations proved that light from these stars was deflected in the direction as predicted by Einstein. Scientists admitted that a deflection was probable, but that this amount would be governed by the orthodox theory. Einstein claimed that the deflection would be twice this amount. At the sun's limb the deflection according to Einstein, must equal 1.75″. Observations found it to be 1.98″, allowing a probable error of 6 per cent.

The sun's attraction is twenty-seven times greater than that of the earth's. Einstein states that the displacement is inversely proportional to the apparent distance of the star from the center of the sun. A star in line with the edge of the disc will have a displacement of 1.75 seconds. This amount approximates the thousandth part of the apparent diameter of the sun. Two English expeditions were dispatched to observe the total eclipse on May 29, 1919. The observers Eddington and Cottingham went to Prince's Island, off the coast of Guinea. The observers Crommelin and Davidson, were stationed at Sobral, Brazil. From seven plates the displacement was found to be as follows: 1.2″, 0.92″, 0.84″, 0.58″, 0.54″ 0.36″, 0.24″. Einstein's would be as follows: 0.88″, 0.80″, 0.75″, 0.40″, 0.52″, 0.33″, 0.20″. In justice to this theory it must be said, that considerable uncertainty attended these observations. The observers at Prince's Island, gave their result at 1.64″.

The discrepancy in the perihelion of Mercury had not been accounted for. Leverrier, found this amount to equal 43 seconds per century. Einstein's figures correspond with this amount. In this way he first verified his theory.

If Einstein's theory is correct, there should in a gravitational field be a displacement of the lines of the spectrum toward the red region. Unfortunately no displacement has been found. There may be some cause yet to be discovered for the failure of the theory. Spectroscopists are pretty well agreed that the theory needs modification.

In working out his theory of relativity, Einstein has ignored the ether.

The earth's motion relatively to the stationary ether should vary at different points on the orbit. No measurable phenomena has been recorded. Motion relative to the ether has never been proven.

Prof. Lorentz, of the University of Leyden, Holland, is given much credit by Einstein for help in working out this theory.

24. Huyghens Wave Theory of Light.—Huyghens, the brilliant Dutch physicist (1629–1695) presented a wave theory that does not only account for light but thermal and electrical phenomena as well. It is well known that light travels through a vacuum and the interstellar space very freely. To account for the transverse wave motion he was obliged to advance the ether theory. Because sound waves and water waves bend readily around corners and light does not it was contended by many that light is not propagated in waves. This objection has been so completely overcome and so many experiments have been made taking the wave theory as a basis that it is now the commonly accepted theory.

It has been demonstrated that the wave length shortens as the rate of vibration increases. Also the effect that an ether wave produces is due to its wave length *i.e.*, a wave that impinges upon the retina and excites the visual act is a light wave. A wave that heats a body is a heat wave. And a wave that produces chemical change in a body is an actinic wave.

The wave length of the extreme red is calculated to be .000760 mm. The wave frequencies at 395 trillion per second. The following table gives the wave length in millimeters, also the vibrations per second for the different colors of the spectrum. The letters *A*, *C*, *D*, etc. are the Fraunhofer lines.

	Length of waves in millimeters	Number of waves per second
Dark red	*A* 0.000760	395×10^{12}
Orange	*C* 0.000656	458×10^{12}
Yellow	*D* 0.000589	510×10^{12}
Green	*E* 0.000527	570×10^{12}
Blue	*F* 0.000486	618×10^{12}
Indigo	*G* 0.000431	697×10^{12}
Violet	*H* 0.000397	760×10^{12}

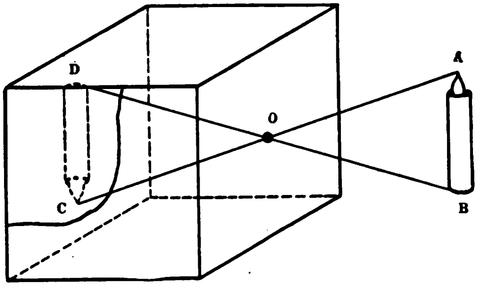


FIG. 15.

Physicists have been able to measure waves between 40,000 trillion and 107 trillion per second.

The above table represents the colors which are found in the solar spectrum and excite the visual centers. The infra red region lies beyond the dark red line and the Ultra violet region lies beyond the Violet line.

25. Rectilinear Propagation of Light.—Light travels in straight lines. It does not bend around corners freely as sound waves. If a beam is admitted into a darkened chamber it will be observed that the beam has sharply defined edges.

Fig. 15 represents a darkened chamber with a small aperture in the wall at O. The rays *A*, *B*, in passing, through cross at the aperture. Hence the inverted image *C*, *D*.

In Fig. 16. Light from the small opening *A*, *B*, is cut off by the body *C*, *D*, and a dense shadow *E*, *F*, is formed upon the screen. This shadow is the ***umbra***.

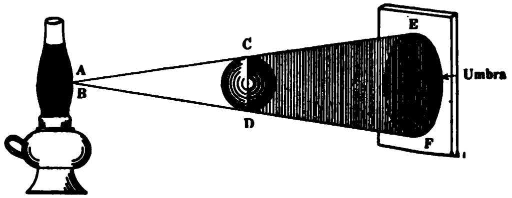


FIG. 16.

Fig. 17 represents a much larger opening. The rays *A*, *B*, determine the outline of the ***umbra*** *E*, *F*. The rays *C*, *D*, produce a lighter shadow *G*, *H*, which is the ***penumbra***.

26. Intensity of Light.—The intensity of radiation is governed by the following laws:

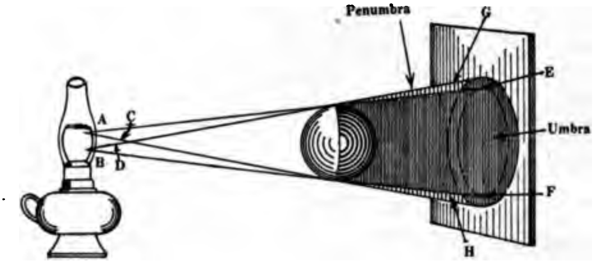


FIG. 17.

First.—Varies inversely as the square of the distance between this surface and the surface of radiation.

Second.—Varies with the angle that the incident radiation makes with this surface; being at a maximum when the surface is perpendicular to the direction of propagation.

Fig. 18 illustrates the first law. *A*, is a small square opening. The plane *B*, twice as large; *C*, four times as large, etc. The planes *A*, *B*, *C*, and *D*, are uniformly spaced. As light proceeds from a source it diverges and spreads over an increasingly larger area.

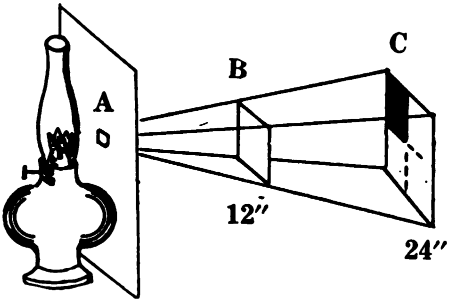


FIG. 18.

27. Photometry.—Photometry is the measurement of the relative intensity of two lights. The photometer, Fig. 19, shows the necessary arrangement. The unit of measurement is a sperm candle of ⅙ pound and burning at the rate of 120 grains per hour. It is known as the standard candle. The intensity of a light is thus expressed in candle power.

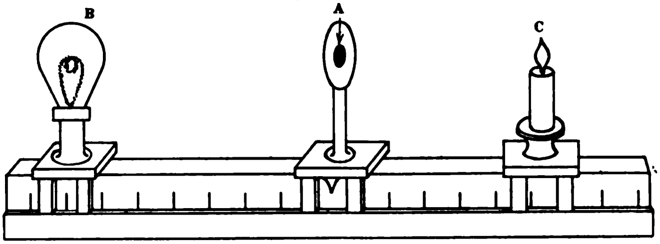


FIG. 19.

In Fig. 19, *A*, is the photometer. It is made of a translucent material. Light from *B*, and *C*, illuminate *A*. *B*, and *C*, are slid along the graduated scale until *A*, appears to be equally illuminated on both sides. The relative intensity is then calculated from the graduations.

28. Spectroscope.—The spectroscope, Fig. 20, is an instrument that enables us to study the different colors mentioned in the preceding table. For a dispersion piece a prism, *P*, or diffraction grating is used. The instrument has a tube *C*, with an adjustable slit in one end, through which the light must pass. This tube is the collimator. At the other end is a plus lens so located that the slit is at its principal focus. Thus waves emerge from the lens parallel. In order to determine the position of certain lines it is sometimes necessary to have another tube *T*, which carries in one end a finely engraved scale on a transparent plate; also a converging lens to render the light waves parallel. These waves are reflected from the nearest face of the prism and enter the telescope with the waves from the other source which are to be analyzed. An eye at the eye piece of the telescope while observing the spectrum sees a magnified image of the scale.

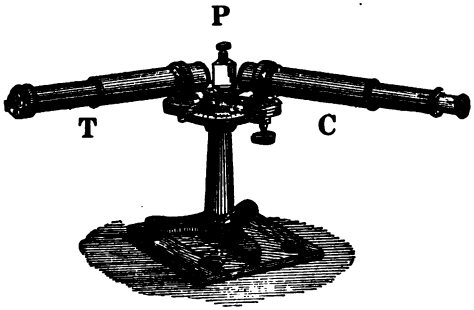


FIG. 20.

29. Solar Spectrum.—It is a well known fact that waves vary in refrangibility; the violet being more refrangible than the red. This can be demonstrated as in Fig. 21. A prism is fixed in a darkened chamber and a beam of sunlight admitted through a slit. This beam passes through the prism and the colors are arranged upon the receiving screen. As might be expected there are no sharp lines of demarkation between the colors. The colors produced by sunlight are as follows: Red, Orange, Yellow, Green, Blue, Indigo and Violet.

30. Recomposition of White Light.—Light may be recomposed if caused to pass through a second prism with its base opposite to that of the first, Fig. 22. Or a concave mirror, Fig. 23, or a convex lens, Fig. 24, may be substituted for the second prism.

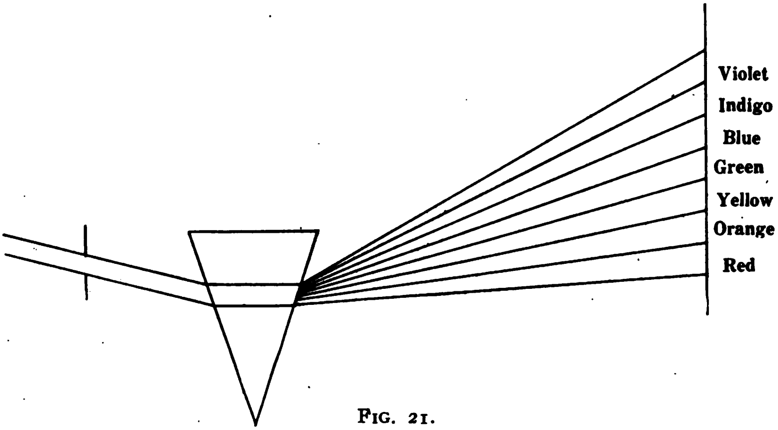


FIG. 21.

31. Fraunhofer Lines.—When sunlight forms a spectrum it will be found crossed by dark lines, Fig. 25. Some of the lines are due to absorption of our own atmosphere, but the majority are due to *selective absorption* of the sun's atmosphere. Aided by this spectrum we may analyze a substance, *e.g.*, the *D*, line indicates sodium in the sun's atmosphere.

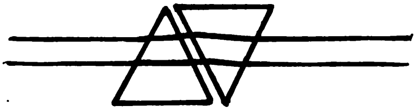


FIG. 22.

32. Laws of Spectra.—*First, incandescent solids and liquids give continuous spectra. Second, incandescent rarefied vapors and gases give discontinuous spectra consisting of colored bright lines or bands.*

Third, if light from an incandescent solid or liquid passes through a gas at a temperature lower than that of the incandescent body, the gas absorbs rays of the same degree of refrangibility as that of the rays that constitute its own spectrum.

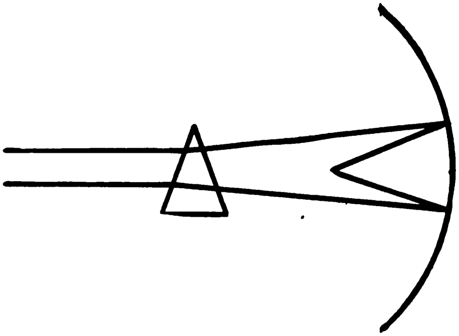


FIG. 23.

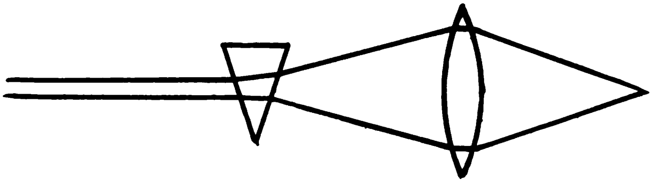


FIG. 24.

33. Color.—A light of but one single color is said to be ***homogeneous*** or *monochromatic*. The only waves that excite the visual act are red, orange, yellow, green, blue indigo and violet. The color of a body is represented by the light which it reflects, the other waves being absorbed.

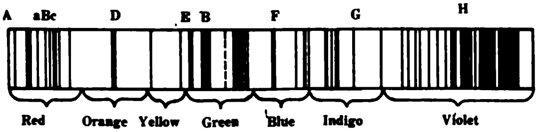


FIG. 25.

Certain substances are possessed of the property of ***selective absorption***, ***i.e.***, waves of a certain length are absorbed, the remainder reflected.

A red substance appears so because it reflects only the red waves. A green piece of glass transmits only the green waves. A white object has no color, because it absorbs none of the incident light which of course is reflected.

34. Complementary Colors.—A combination of colors which when superimposed produces the sensation of greyish white are said to be complimentary. Following are complementary combinations: Red and Bluish Green. Orange and Blue. Yellow and Indigo. Yellowish green and Violet. Green and Purple. This may be demonstrated. Arrange the colors in V shape, upon a disc, Fig. 26, and rotate it rapidly. These colors will produce the visual sensation of greyish white. This is because, first each color produces a different visual sensation. And second, that the sensation is of longer duration (about 1/10 of a second) than the impression producing it. And third, the impression of all the colors have been made before the sensation of any have disappeared. It might be said that one sensation has been overlapped by the other.

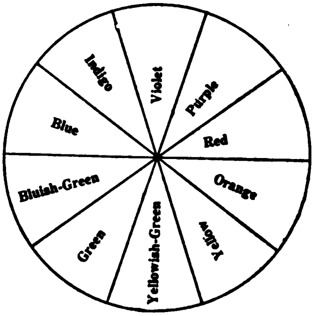


FIG. 26.

35. Mixing Pigments.—In mixing pigments we cannot obtain the same results as in mixing colors. This is because one pigment in part absorbs the other. The resultant color represents that which escaped absorption.

36. Chromatic Aberration.—*Dispersion. Unequal refraction of the various waves results in the border of the image being fringed with different colors.*

The diagram, Fig. 27, illustrates the more refrangible violet rays coming to a focus at *B* sooner than the red rays which focus at *C*.

If a screen was placed at *A*, the margin of the image would be of a reddish tint. Because the red rays have not focused. If placed at *D*, the margin would be of a violet tint. Because the violet rays have crossed at *B*, and are now diverging.

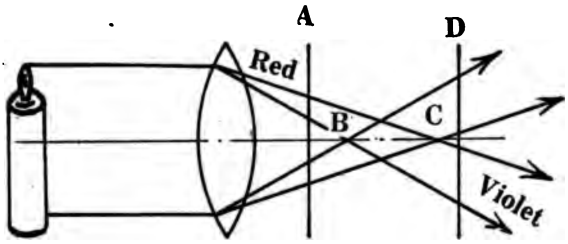


FIG. 27.

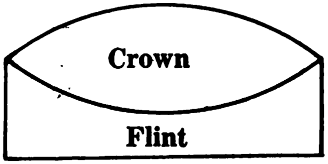


FIG. 28.

Chromatic aberration is overcome in the *Achromatic lens*. A biconvex lens of crown glass being combined with a plano-concave lens of flint glass. Fig. 28.

37. Spherical Aberration.—*Rays that have passed through the peripheral and central parts of a lens do not focus at one point.*

Rays passing through the peripheral portion of a spherical lens focus at a shorter distance from the lens, than those passing through the central portion. The periphery of the image is blurred. Or if the periphery is in focus the central part of the image is blurred.

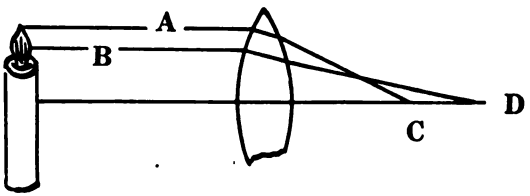


FIG. 29.

Fig. 29 illustrates the peripheral ray *A*, coming to a focus at *C*, which is at a shorter distance from the lens than the more central ray *B*, which focuses at *D*.

38. Spherical Aberration in lenses and mirrors is lessened by using a diaphragm which cuts off the peripheral part of the medium. As some light is shut off the image will not be so bright, although better defined. The iris of the eye acts as a diaphragm. Spherical aberration is corrected in the aplanatic lens.

Spherical aberration in mirrors is overcome by decreasing the curvature from the vertex outward. Such mirrors are designated *Parabolic Mirrors,* Fig. 30. They are used in locomotive head lights, light houses, astronomical telescopes, etc.

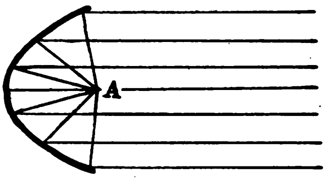


FIG. 30.

Fig. 30 illustrates how the rays are reflected to a common focus at *A*. In Fig. 30 the rays are reflected to the point *A*, this mirror having a parabolic curve.

39. Interference of Ether Waves.—In presenting his wave theory Huyghens claimed that each point of a wave front must be taken as the center of a disturbance which is propagated in all directions, through the medium. In Fig. 31 let *A*, *B*, be a wave front, passing through a slit and moving toward *a*. Let *b*, be a point so situated that its distance from *A*, is greater than its distance from *B*, by half a wave length. This half wave length is represented by the distance *A*, *C*. According to Huyghens a disturbance from *A*, will travel to *b*, in the same length of time required for a disturbance

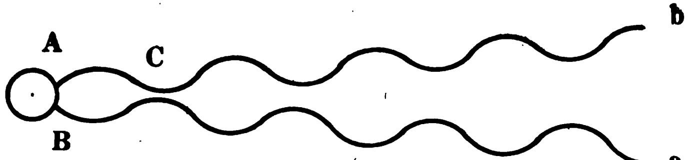


FIG. 31.

from *B*, which starts a *half period* later. The displacement at *b*, caused by the disturbance at *A*, will be equal, but opposite to that from *B*. Thus the two points *A*, *B*, will neutralize each other.

In 1801 Dr. Young made some valuable experiments with light waves. He admitted a beam of sunlight into a darkened chamber by means of a narrow aperture. Over this aperture he placed a screen which had two small openings close together. The light was then received upon a white screen. The result was a series of alternating bright and dark bands. These bands disappeared when one hole was covered. Young claimed that this phenomena was due to interference of light waves.

Fig. 32, *A*, represents a source from which two waves *a*, and *b*, are propagated. The vibration of these two waves is identical. Thus they reinforce each other. In Fig. 31, *a*, *b* differ in phase and it can be seen that they interfere, which of course means destruction more or less complete.

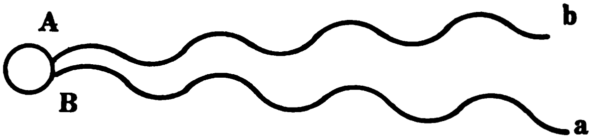


FIG. 32.

Grimaldi long before (about a century and a half) had observed light and dark fringes about the shadows of a small opaque body. Newton in defense of his emission theory claimed that the light particles are attracted or repelled as they come near the edge of bodies.

The beautiful color of soap bubbles is due to interference of waves which differ in phase. This phenomenon cannot be accounted for by the emission theory.

If two pieces of plate glass are clamped together at the centers and an eye looks obliquely through them a beautiful play of colors will be seen surrounding the point of greatest pressure. Under the influence of *monochromatic light*, yellow bands separated by dark bands will be seen.

Newton observed that if a plano-convex lens is pressed against a plate glass, Fig. 33, an eye looking downward will see a number of rainbow bands. These bands are known as *Newton's rings.* This phenomena is explained as follows: When light passes through the lens, some is reflected from the curved surface; other waves pass through and are reflected from the plate. These latter waves it can be seen must traverse the wedge-shaped air film twice. When the thickness of the air film at any point is such that the two sets of reflected waves unite in opposite phases interference must result. If this phenomena is observed by white light interference takes place between the red waves. Instead of a red band we will have one represented by the complementary color of red, which is green. If the incident light is red a dark ring will appear at the same distance. At another distance the violet waves interfere and the band in its place is represented by the other constituent waves of the incident light.

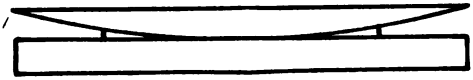


FIG. 33.

40. Diffraction.—Newton always to the defense of his emission theory claimed that according to the wave theory light would bend around corners very readily, as sound waves. Huyghens explained that when light and sound waves pass through an aperture that secondary systems of waves pass onward. In the case of sound waves passing through an aperture much less than the wave length that the secondary waves will suffer but slight interference. With light waves if the aperture is very large as compared with the wave length the secondary waves will interfere and destroy each other. Light passes through an aperture in straight lines where sound waves are diffused in all directions. When light waves pass through a very small aperture or slit, and the image of the slit is received upon a screen the margin will be blurred by color fringes.

Newton's emission theory is of no avail in accounting for diffraction.

41. Diffraction Gratings.—Diffraction gratings are made by ruling very fine parallel lines with a diamond point. If glass is the substance used the fine diamond lines are almost opaque and the waves pass through between these lines.

Reflection gratings are produced by ruling very fine lines on a highly polished speculum metal. A spectrum of great purity may be obtained if light passing through a slit is allowed to fall upon the grating. Prof. Rowland ruled 160,000 lines in a space of six inches on a concave spherical mirror. The solar spectrum was photographed in sections, which when placed end to end extend over a length of thirty feet. With the aid of this device the measurements of wave length are considered the most accurate to date.

42. Iridescence is a phenomenon observable when light shines on mother of pearl, peacock feathers, spider webs, etc. The color or lustre seems to be changed according to the nature of the reflecting surface and direction of the light. The nature of the colors is due to the structure of the surface. This fact has been demonstrated as follows. An impression of a piece of mother of pearl was made in white wax. This wax would exhibit the same color effect as the mother of pearl. If the source of illumination is very large there may be only a change in sheen and the colors almost absent.

43. Measuring Wave Length.—In the excellent treatise "A First Course in Physics," by Millikan & Gale, is set forth a simple method of measuring the wave length of a certain color wave. Two pieces of plate glass about one-half inch wide by four or five inches long are arranged to form a wedge. Let the incident light be yellow. Light and dark bands will appear across the plate. Assume that the air wedge is 10 centimeters long. At the vertex the plates are touching. At the base of the wedge the plates are separated .03 millimeter.

The dark bands are 1 millimeter apart. The thickness of the wedge at two consecutive dark bands 1 millimeter apart must be

.01 of its thickness at the base, *i.e.*, 1/100 of .03 millimeter or .0003 millimeter. Since it is the double path through the air wedge which must be exactly one wave length longer than the double path, the thickness of the air wedge at these points must differ by ½ wave length. It follows that the wave length of yellow light must be 2 × .0003 = .0006 millimeter.

To find the number of vibrations per second divide the velocity by the wave length. The velocity being taken at 30,000,000,000 centimeters per second and wave length at .00006 centimeter, the number of vibrations per second equal 500,000,000,000,000 (500 trillion).

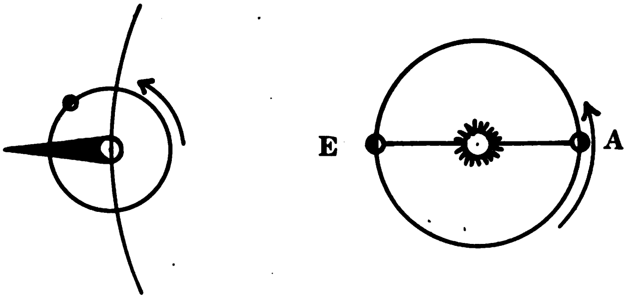


FIG. 34.

44. Measuring the Velocity of Light.—In 1675 Olaf Roemer, a young Danish astronomer, fixed the velocity of light at 186,000 miles per second, Fig. 34. He observed that the nearest of Jupiter's moons passes within the shadow of that planet at regular intervals. These intervals are not equal when observed from the earth, being longer when the earth travels from *E*, to *A*, than from *A*, to *E*. It requires 16 min. 36 sec. for light to pass over the diameter of the earth's orbit.

45. Fizeau's Method.—Fig. 35. The arrangement is very simple. *L*, is the source of light. *M*, and *M′* are mirrors. *W*, is a toothed wheel so arranged that a tooth will pass through the line *M*, *M′*, and cut off the light. If the wheel is at rest and an open space comes in line with M, M' a bright spot in the mirror M' will be observed. This spot is an image of L, reflected by M. If the wheel is rotated above a certain velocity a wave will be reflected through an opening to M' and back in time to be stopped by a tooth. If the velocity of the wheel number of teeth and distance between stations is known the speed of light may be easily calculated. Let d, equal the distance between stations. The wheel will have to make r, revolutions to cut off the returning light. Let t, equal the number of teeth in the wheel. The teeth and blank space are of equal width. Then

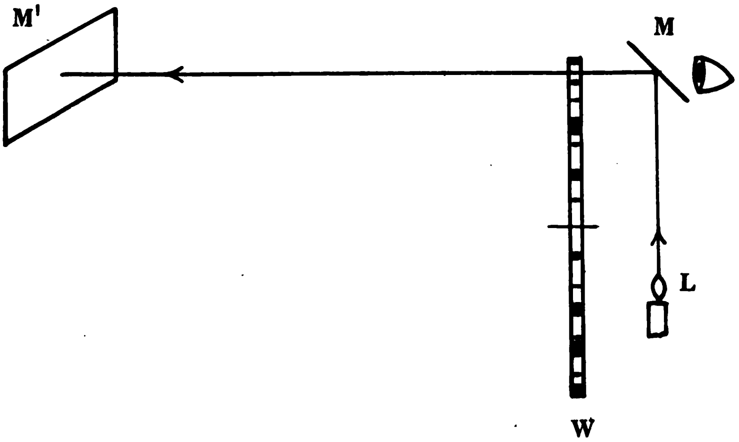


Fig. 35.

$$V = 4dtr.$$

The different color waves apparently travel at the same speed. Were it not for this we would observe a change of color after the eclipse of Jupiter's moons.

46. Velocity in Different Media.—It is a well established fact that the velocity of light is not the same in all media. This is due to a property exhibited by all transparent substances and expressed, *index of refraction.* Experiments have demonstrated that the higher the *optical density* the slower it is traversed by light waves. To illustrate this we will assume that a wave from O', Fig. 36, is passing through the denser medium water, to the rare medium air. If the velocity was not changed, the wave front when passing out would continue to expand as indicated by the heavy circles. After a length of time the wave front would be at A, and the object appear to be at O', but instead the wave front after emerging into the air has gained in speed and is at B. The object appears to be at O. This phenomena accounts for the apparent shoaling of a body of water.

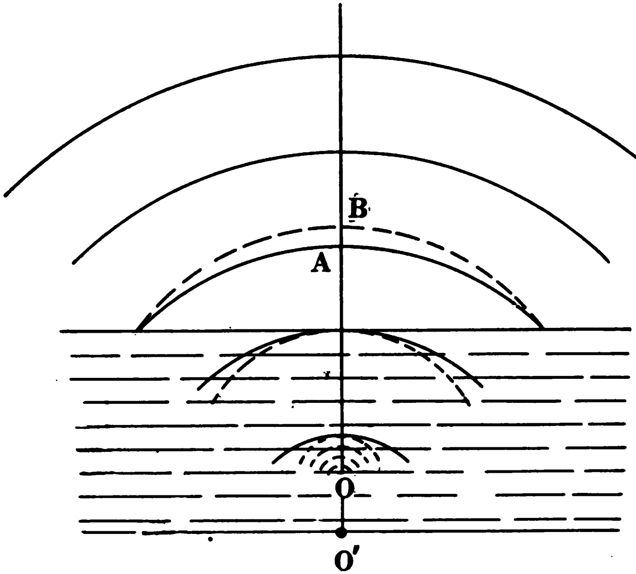


FIG. 36.

If an eye looks vertically into a glass jar of water the bottom will appear to be raised.

If the apparent location of the bottom is marked with a finger it will be found to be raised about ¼ the height of the jar.

47. Index of Refraction.—The velocity of light waves decrease as the optical density of a medium increases. The index of refraction equals the sine of the angle of incidence divided by the sine of the angle of refraction, thus

$$\text{Index of refraction} = \frac{\text{sine of the angle of incidence}}{\text{sine of the angle of refraction}}.$$

Unless more exact calculations are desirable we may neglect the index of gases. The index for light passing from air to water is 1⅓, etc. The following is a table of *Absolute Indices:*

Air at 0°C. and 760 mm. pressure	1.000294
Pure water	1.33
Alcohol	1.37
Spirits of turpentine	1.48
Canada balsam	1.53
Crown glass approx	1.53
Agate	1.54
Flint glass approx	1.61
Diamond approx	2.5
Lead chromate	2.97

By the absolute index of refraction we mean that light is passing from a vacuum into the medium in question.

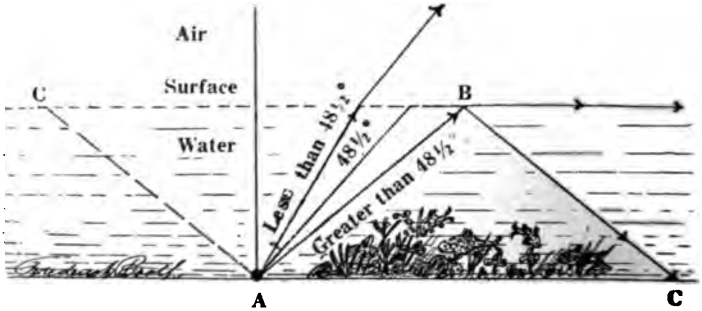


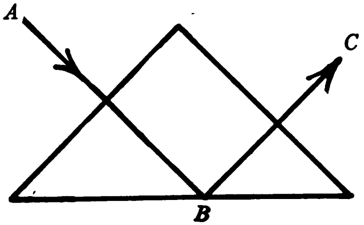
FIG. 37.

48. Critical Angle or Limiting Angle.—The critical angle is best explained from the diagram, Fig. 37. A ray passes from the denser medium water at such an angle that when it comes in contact with the rarer medium air, it is so refracted that it skims the surface.

The incident ray travels at an angle of 48.5°, the critical angle for water. This angle may be taken as the dividing line between reflection and refraction. From the diagram it can be seen that if the angle of incidence was greater the ray would strike the surface, which acts as a plane mirror and be reflected back into the water, undergoing *total internal reflection.* If the angle of incidence was less the ray would pass out into the rarer medium air, and be refracted. The index of refraction determines the critical angle.

Following is the approximate critical angle for different media Flint glass 38.6°. Diamond 23.6°.

An observing eye at *A*, sees objects in the air that come within the cone *A*, *B*, *C*. Outside of this cone the observing eye would see objects lying upon the bottom of the pond by internal reflection, *e.g.*, rays from an object at *C*, strike the surface at *B*, and are reflected back to *A*.



48½° Greater than the Critical Angle

FIG. 38.

49. Total Internal Reflection by a Prism.—The prism, Fig. 38, forms a right angle isosceles triangle. The ray *A*, strikes the first surface normal and on passing through comes in contact with the other surface, the hypothenuse at *B*, which is at an angle of 45°. This being greater than the critical angle the surface acts as a plane reflecting mirror. Here the ray cannot pass out, but is reflected in the direction of the third surface *C*. The original course being changed by 90°.

This prism makes the most perfect reflector where the course of the light is to be changed by 90°. For this reason they are used in higher class optical instruments.

50. Polarization of Light.—When the waves that make up a beam of light are shut off excepting in one plane, the beam is said to be polarized. In Fig. 39, let *A*, represent a vertical wave and *B*, a horizontal wave. *C*. and *D*, are tourmaline crystals with the axes right angles. The wave *A*, passes through *C*, because it is at the same plane as the axis of *C*; the wave *B*, is stopped by *C*, because it lies in a plane at right angles to *C*. The axis of the second tourmaline plate *D*, is at right angles to the wave *A*, for this reason *A* is now stopped as no light passes through *D;* darkness is the result.

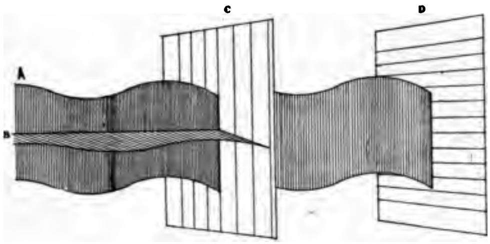


FIG. 39.

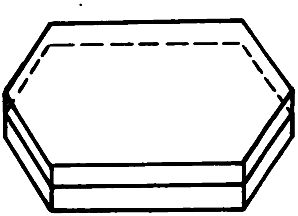


FIG. 40.

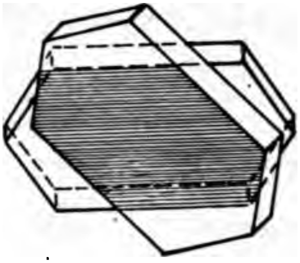


FIG. 41.

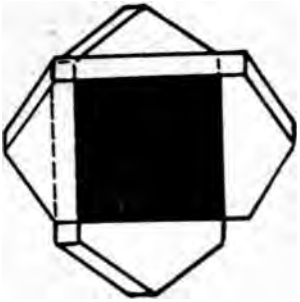


FIG. 42.

If two tourmaline plates are placed with the axes together as in Fig. 40, the light will pass through freely. If rotated as in Fig.

41, a portion of the light will be stopped. If rotated so that the axes are at right angles, Fig. 42, practically none of the light will pass through. The second plate, of course, stops the light.

51. Polarization by Reflection.—A good way to demonstrate polarization by reflection is to admit a small beam of light into a darkened room through a small round opening made in a shutter. With a mirror the beam may be reflected in any direction. But if the mirror is laid upon a table so that the beam impinges at a certain angle, a remarkable change will occur. Light reflected by this mirror may be reflected up or down by another mirror, but not to the right or left. The light is said to be polarized in a *vertical* plane.

If the mirrors are so arranged that the beam may be reflceted in the horizontal meridian the plane of polarization is said to be *horizontal.*

The plane of polarization by reflection is always perpendicular to the reflection surface.

To obtain the best results the angle of incidence for both plates should be 56¾°.

Let *L*, Fig. 43, be a wave from a candle incident upon the mirror *M* at *a*. *L*, is reflected by the second mirror *M'* at *b*, and an image of the candle situated at *L*, will be observed by the eye at *E*. It can be seen that the plane *L*, *a*, *b*, *E*, is perpendicular to the reflecting surfaces.

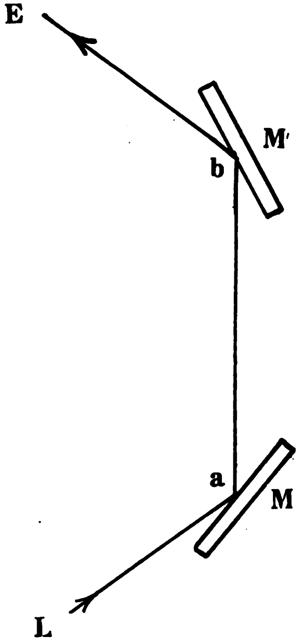


FIG. 43.

The arrangement in Fig. 44, is similar to that in Fig. 43, and an eye at *E* also sees an image of the candle *L*.

Fig. 45 represents the mirrors arranged in planes that form a right angle. The wave *L*, impinges at *a*, and is reflected to *b*, where it is lost. An eye at *E*, therefore, would not see an image of *L*.

The correct angle for polarization varies for different media. For water slightly greater than 53°. For sulphur nearly 64°. Diamond 68°.

52. Polarization by Double Reflection.—A great many years ago it was discovered that certain substances possessed the peculiar property of causing small objects to appear double. This property belongs chiefly to crystallized bodies, such as Iceland spar (crystallized carbonate of lime), quartz, etc. A piece of glass that has been heated and then cooled suddenly acquires this property.

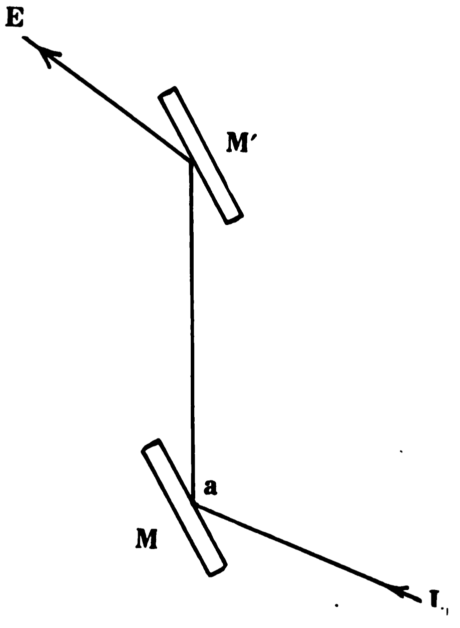


Fig. 44.

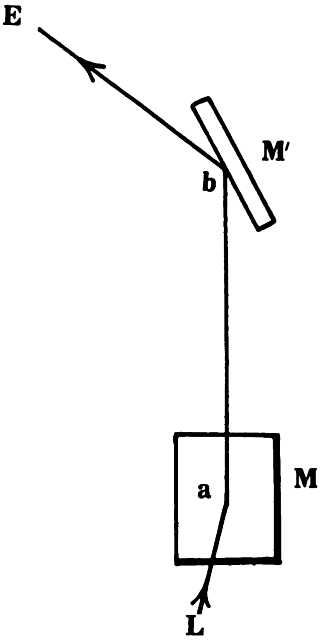


Fig. 45.

Let Fig. 46, *A*, *B*, *C*, *D*, *E*, *F*, *G*, and *H*, represent a block of Iceland spar. *R*, is a wave perpendicular to the surface, and on entering at *K*, it will divide. One part will pass straight through to *I*, and not suffer refraction. The other will be refracted to *J*. The ray *K*, *I*, is called the *ordinary wave.* The wave *K*, *J*, is the *ex-*

traordinary wave. If a piece of paper upon which a dot is made is placed under the crystal, there will appear two dots. One at *I*, and another at *J*. If the crystal is now revolved on *I*, the dot *J* will, appear to revolve around *I*.

If a very thick piece of Iceland spar is placed over an object, *e.g.*, a printed word, Fig. 47, the word will appear double. The thicker the crystal the greater will be the separation.

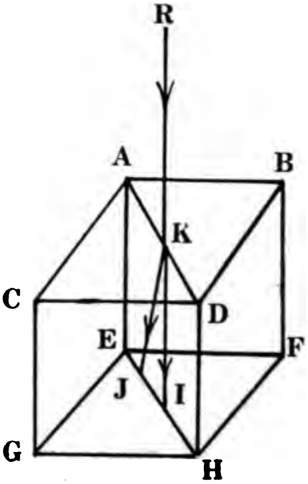


FIG. 46.

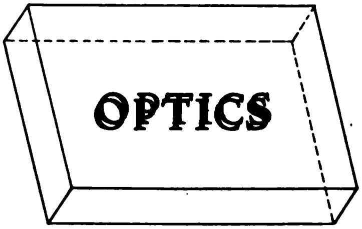


FIG. 47.

A wave may pass through a crystal in one direction and not suffer double refraction. This course is known as the *optic axis* or *axis of double refraction.*

Let Fig. 46 represent a crystal of Iceland spar, *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*. The *axis* is in the direction of the line *A*, *H*, which joins the two obtuse solid angles. A section through *A*, *D*, *H*, represents the *principal section.*

Certain substances possess a number of axes of double refraction, but a discussion will not be attempted in this treatise.

When the wave which has suffered double refraction is examined by the polariscope it will be found that the waves are polarized in planes at right angles. It is interesting to note that these two waves do not represent different color as waves separated or decomposed by a prism.

53. Colors Produced by Polarized Light.—The arrangement in Figs. 43, 44 and 45 will answer, only it is necessary to interpose a piece of thin mica or selenite between the reflecting surfaces *M* and *M'*. As *M'* is revolved on a vertical axis there will be one point where the mica exhibits a faint red. As *M'* is revolved the red increases to the highest brilliancy. As *M'* is revolved the red diminishes until it disappears when a faint green of gradually increasing brightness appears. It then gradually fades away. The resultant colors depend upon the thickness of the mica and are said to be *complementary to each other*, *i.e.*, combined they will produce white light.

If *M'* is replaced by a prism of Iceland spar both colors will be observable at the same time. At the period of greatest brilliancy if they are made to overlap, white light will result.

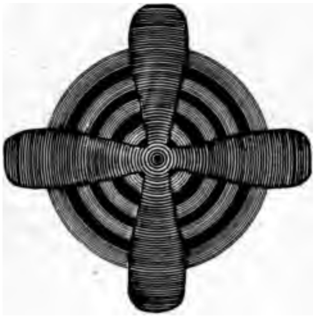


FIG. 48.

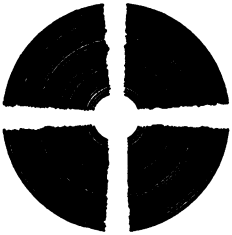


FIG. 49.

If the mica varies in thickness each portion will display a tint depending upon the cross sectional area. These plates are perfectly transparent, but when observed by polarized light they display colors gorgeous beyond description.

If tourmaline plates are held between *M* and *M'*, Figs. 43, 44 and 45, when the reflecting surfaces are in the position, Fig. 43, we will see the ring and cross in Fig. 48. When *M'* is revolved the second ring and cross, Fig. 49, appears. The colors which appear in one ring are complementary to those of the other.

54. Atmospheric Refraction.—The earth is surrounded by an atmosphere that gradually decreases in density and refractive power.

In Fig. 50 waves from the sun enter our atmosphere and are bent in a curved line of increasing sphericity. To an observer at *A*, the sun *S*, appears to be at *S′*. The sun is visible when its true location is beneath the horizon.

55. The Mirage.—When the condition of the atmosphere is such that the waves *A*, *B*, Fig. 51, pass upward and are then refracted downward in the path *B*, *E*, the object will appear to be at *C*, *D*, fainter and above its true position.

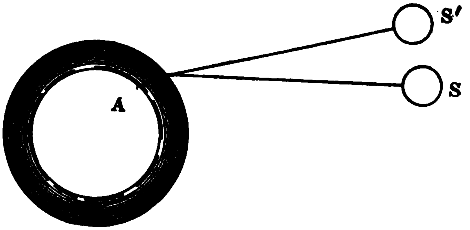


Fig. 50.

In Fig. 52, the waves *A*, *B*, are refracted downward and as the object always appears to be in a direction coincident with the wave as it enters the eye, the apparent location of the object is at *C*, *D*.

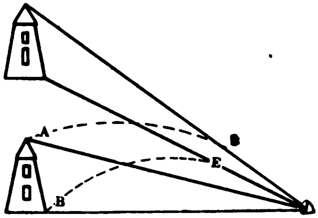


Fig. 51.

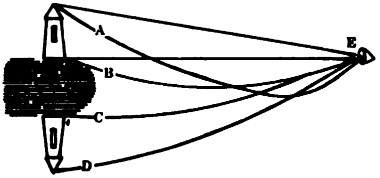


Fig. 52.

56. The Rainbow.—When a rainbow appears it will be noted that it and the sun are on opposite sides of the observer.

Let Fig. 53, represent a drop of water suspended in the air. *S*, *A*, is a wave from the sun. It enters at *A*, is reflected at *B*, and again refracted at *C*, where it passes out and enters the eye *E*. If we can imagine *S*, and *A*, to be a number of waves, also that these waves vary in refrangibility that of the violet being greatest, it is evident that *E*, will observe the primary colors of the rain bow, although to do this it is necessary to change the position of the eye.

Fig. 54, shows how the *secondary bow* is formed. A wave from the sun *S*, impinges upon the drop at *A*, is refracted to *B*, refracted to *C*, *D*, and emerges passing on to the eye at *E*. It can be seen that the colors will have a reverse arrangement.

57. Radiant Heat.—When ether waves impinge upon a substance some are reflected and the remainder absorbed. Certain substances have the property of absorbing practically all of the incident waves, while others reflect most of the waves. To illustrate this statement, two thermometers are chosen, one bulb is coated with lamp black, and the other bulb not so treated. It will be observed that if both are placed in a beam of light that a rapid rise in temperature will be noted in the coated thermometer, and scarcely any change in the other.

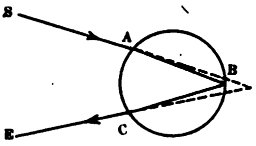


FIG. 53.

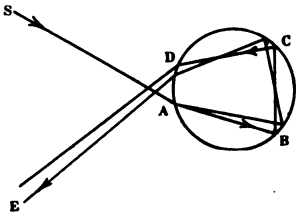


FIG. 54.

This demonstrates that lamp black absorbs heat waves, also that a polished surface reflects most of the waves. From this demonstration it can be seen that good absorbers are poor reflectors (and *vice versa*).

The ultra violet waves when absorbed by a plant produce chemical changes and thus contribute to plant growth and color. A potato plant growing in a dark cellar is possessed of a pale green color because it has been deprived of the actinic action.

Thus it can be seen that a body is warmed through its property to absorb ether waves. The effect that an ether wave produces in a body depends more upon the absorbing property than wave length, *e.g.*, a wave that produces the visual sensation will when absorbed, be productive of heat to some extent. Air is not the medium by which heat is conveyed, as heat is transmitted very readily through the most perfect vacuum. Heat waves do not warm the air through which they pass to any perceptible degree. If heat was a train of heat units or atoms, it apparently would warm the atmosphere and to a large measure be absorbed.

Heat waves may be reflected and refracted, the same as light waves. It travels at the same rate as light. This statement can be proven during a solar eclipse, light and heat arriving simultaneously. Heat waves are found in the ultra violet region but not in large quantities.

Heat is transmitted by three known modes, *i.e. conduction, convection and radiation.* Conduction implies heat that is transmitted from one particle to another in a body. This process is slow. Convection means that the heat is conveyed by some agent. This occurs in heating a building with water pipes. Radiation has already been pointed out.

58. Reflection of Radiant Heat.—The long heat waves obey the same laws of reflection as light. Leslie proved that the best reflectors are the poorest absorbers. Taking brass as a standard, he prepared the following table of relative reflecting powers:

Brass	100	Lead	60
Silver	90	Amalgamated iron	10
Tin	80	Glass	10
Steel	70	Lamp black	0

The following table is the absolute reflecting power of different substances:

Silver	97	Steel	82
Gold	95	Zinc	81
Brass	93	Iron	77
Platinum	83	Cast iron	74

59. Absorption of Radiant Heat.—Lamp black is stated as the best absorber, as it does not reflect any sensible portion of the incident heat. The ratio of the quantity of heat reflected to that of lamp black in the same length of time, the surfaces being of an equal dimension, represents the emissive power or emissity. Tyndall coated a Leslie cube with different substances and was then able to prepare a table as follows:

	Absorbing power	Emissive power
Rock salt	0.319	0.307
Fluorspar	0.557	0.589
Red oxide of lead	0.741	0.707

60. Refraction of Heat Waves.—Melloni discovered that rock salt transmits non-luminous waves with great facility; while other transparent substances are comparatively good absorbers. He demonstrated that these waves could be refracted to a focus by a rock salt lens. Also that these waves are of less refrangibility than the luminous waves. The refrangibility also decreases with the temperature of the body.

Forbes found the index of refraction for several sources to be as follows:

Mean luminous waves	1.602
Heat from incandescent platinum	1.572
Heat from a lamp without a chimney	1.571
Heat from brass at 370°C	1.568

	Absorbing power	Emissive power
Oxide of cobalt	0.732	0.752
Sulphate of iron	0.824	0.808

In 1912 VonBayer and Rubens measured infra red waves as long as .3 centimeters which is about 400 times the length of the longest visible waves. The longest waves visible in the spectrum are approx. .00008 centimeter.

Infra red waves may be detected by a delicate instrument known as the radiometer.

61. The Radiometer.—Fig. 55, *R*, indicates the Radiometer which consists of a partially exhausted bulb, several aluminum or mica vanes blackened on one side and polished on the other. These vanes are carried by a pivot, which allows free rotation. When this instrument is placed in a beam of light or close to a lamp the vanes will rotate; the blackened side away from the source of radiation. This rotation is due to the blackened surfaces absorbing more than the polished sides. The adjacent air being brought to a higher temperature it exerts a greater pressure.

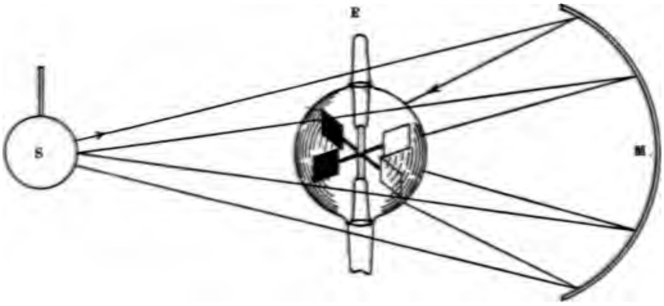


FIG. 55.

Let *S*, be a heated body, *R*, the Radiometer, and *M*, the concave reflecting surface. It can be seen from Fig. 55, that the waves are brought to a focus on the vanes. As the temperature of *S*, is increased so also will the rotation of *R*. If *S*, is a very cold body the vanes will rotate in the opposite direction, because the radiation from the polished surfaces is greater.

62. Polarization of Heat.—Malus, and Berard, demonstrated that heat waves are polarized by the same methods as light waves. Forbes, polarized heat by using tourmaline plates. It made no difference whether the source was a lamp or a piece of brass heated below the luminosity point. The heat was extinguished when the plates were crossed. He took some mica plates split by heat and demonstrated that heat can be polarized by reflection and refraction. These mica plates are almost opaque to light waves but transmit dark heat waves very freely. This is also true of thin hard rubber sheets.

63. Diathermanous and Athermanous Bodies.—A body that transmits radiant heat is said to be diathermanous. Bodies that absorb it are athermanous. It follows then that a substance transparent to light waves is opaque to the non luminous heat waves. Glass is claimed to be slightly transparent to waves even beyond the violet region.

Melloni, demonstrated that a sheet of glass 2.6 mm. thick is opaque to the waves from a blackened copper body at 100°C. It transmits but 6 per cent. at 390°.

Waves from the sun pass very freely through a window glass. The glass is heated slightly. Yet objects in the enclosed room are brought to a comparatively high temperature. This phenomena can be accounted for by a previous statement that the resultant heat in a body is due more to its absorbing qualities than wave length. Thus the light waves traverse the window glass and after being absorbed by the interior bodies are the cause of longer heat waves being radiated from different objects. The glass being opaque to these latter waves, they are caged in and the result is a rise in temperature of the room.

64. Electro-magnetic Theory of Light.—In the year 1865 Maxwell, advanced the theory that light is due to electro-magnetic disturbances in the ether. This disturbance he fixed as conditions of strain and release. He claimed that light waves are electro-magnetic waves.

As strong proof of this theory we know that the electrical discharges of a Leyden jar are oscillatory and excite ether waves that have almost the same speed as light waves. They were first measured by Hertz in 1888. In kilometers it is expressed 300,000 per second. In instantaneous photography the wave length is expressed $\frac{300,000,000}{10,000,000} = 30$ meters. Velocity of light equals 300,000,000 meters per second. Wave length equals velocity divided by the oscillations per second. Electrical waves of .3 centimeter have been measured. As further proof of the undulatory theory of light it is possible to reflect, refract, polarize and cause interference of these electrical waves. In reflecting the waves a metal mirror is used.

65. Cathode Rays.—Fig. 56, represents an apparatus devised by Sir William Crookes, in 1879 to illustrate the following experiment. As the air is being exhausted from this tube an electrical discharge is forced through. When the pressure equals about .01 mm. of mercury a greenish fluorescence appears on the walls about the cathode. As the exhaustion is continued the discharge through the gas disappears, and the walls possess a beautiful fluorescence. This disturbance is assumed to be due to the cathode and is normal to its surface. This is proven by the shadow of the cross *B*.

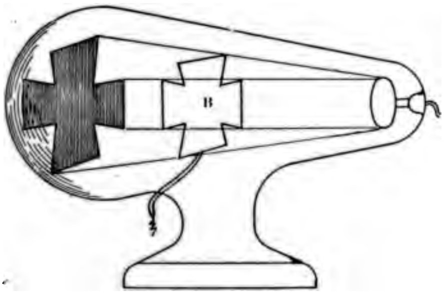


FIG. 56.

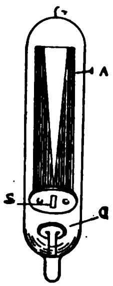


FIG. 57.

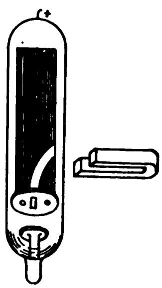


FIG. 58.

Crookes, contended that this disturbance was not of an ether wave form but rather particles projected at a high velocity from the cathode. Crookes in 1879 substantiated his proof by applying a magnet to these rays. Fig. 57 represents the course of the rays. Fig. 58 illustrates these rays being deflected by the magnet as negatively charged particles would behave. *C*, is the cathode. The rays emitted therefrom pass through the slits *S*, and produce fluorescence in the zinc sulphide screen *A*.

In 1893 Lenard, of Germany, produced some experiments that did much to upset the projected particle theory. Lenard, demonstrated that the cathode rays may pass out through the thin aluminum window, and continue for several centimeters in the atmosphere before suffering absorption. Fig. 59, represents the instrument, *C*, the cathode, *A*, the anode, *O*, the window and *E*, the cathode rays.

These rays cannot be gaseous, otherwise they would be stopped by the aluminum. But suppose we imagine these projected particles to be of such minute dimensions that they can pass between the molecules and atoms of ordinary matter, then the corpuscular theory holds good. A remarkable demonstration was made by Lenard, and J. J. Thompson, in 1897. The cathode rays were deflected by an electrostatic field just as we might expect if the particles are negatively charged. The deflection of different magnetic fields being known, and aided by mathematical analysis the mass of the projected particles was found to be $\frac{1}{1800}$ of the smallest known atom which is the hydrogen atom. In 1896 Perrin, a Frenchman, discovered that these particles impart a negative charge to objects upon which they impinge.

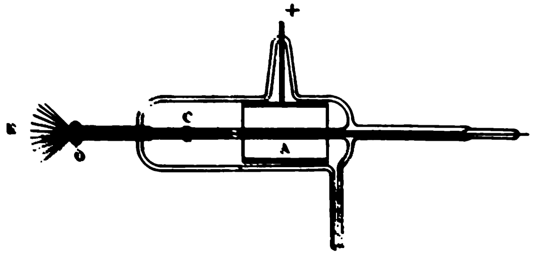


FIG. 59.

The wave theory has been abandoned, far as the cathode rays are concerned.

Hertz, by a remarkable experiment demonstrated that if the

Crookes tube was covered with thin flakes of gold leaf that it would phosphoresce. He also proved that these gold leaf flakes were more transparent to the cathode particles than sheets of mica of the same thickness.

Lenard, in 1894 suspended a photographic plate in the air before the aluminum window, and observed a chemical change.

The speed of the cathode particles has been fixed at 3×10^{10} cm. per second. This velocity lies between one-third and one-tenth that of light.

If the cathode rays are brought to a focus by a concave reflector in the tube they are productive of heat sufficient to bring platinum foil to a white hot color.

66. X-rays.—Rontgen, in 1895, discovered that as the cathode rays impinge upon the walls of the *X*-ray tube or an object placed inside of the tube, that another form of radiation is produced. These rays are known as the *X*-rays or Rontgen rays. A thick piece of platinum is a target for the cathode rays. The rays from the concave cathode focus upon the center of the platinum. From this point the *X*-rays are propagated in all directions.

Like cathode rays *X*-rays cannot be refracted or polarized. *X*-rays are productive of fluorescence. They penetrate substances which are impervious to cathode rays. Glass is quite impervious to cathode rays but not *X*-rays. They are not deflected by a magnetic or electrostatic discharge. The conclusion must be drawn that they are not streams of electrically charged particles.

The nature of the *X*-ray is not understood, but they are thought to be an ether disturbance caused by the collision of the cathode particles when they strike the glass wall. These rays, like the cathode rays, will cause an electrified substance to gradually lose its charge.

The *X*-rays will pass through wood, card board, cloth and flesh. Bones are quite opaque to these rays. *X*-ray photography is a very valuable branch of medical science.

67. Radioactivity.—Becquerel, in 1895 wrapped a photographic plate in a piece of black paper and placed a coin upon the paper.

Above the coin was suspended a quantity of mineral uranium. This outfit was placed in a dark room for several days. When the plate was developed a shadow of the coin was found similar to an *X*-ray picture. This experiment proves that uranium emits a form of ray that penetrates opaque bodies and affects a photographic plate similar to that of the *X*-ray. Uranium particles travel at a velocity greater than that of cathode rays and about half that of light.

68. Alpha and Beta Rays.—Rutherford, in 1899 proved uranium rays to be of two kinds, Alpha and Beta rays.

The Alpha rays are not so penetrating but have a greater ionizing effect. They are recognized as bodies positively charged, of atomic size, and in mass equal to that of the helium atom or hydrogen molecule. A layer of air three or four centimeters thick will absorb them. Passing through a strong magnetic field they are slightly deflected and in a direction opposite to that of the Beta rays.

The Beta rays are treated as projected electrons, *i.e.*, cathode rays.

69. Gamma Rays.—Radium and actinium produce Alpha, Beta and another type known as the Gamma ray. The Gamma ray has a penetrating power estimated at a hundred times greater than the Beta rays. It is not deflected by electric or magnetic fields.

70. Optical Signs and Abbreviations.

Degree °.
Infinity, Distance 20 ft. parallel rays. $\propto$
Left Eye *L.E.*
Minus, Concave —.
Near point, Punctum Proximum *P.P.*
Plus, Convex +.
Prescription *RX.*
Right eye *R.E.*
Spherical *S.* or *Sph.*
Vision *V.*

71. The Dioptric System.—About the year 1860 the inch system was in vogue. A lens of some number corresponded with the focal length, *e.g.*, a number 5 glass had a focal length of 5 inches. As an inch is not the same in all countries much confusion resulted. The English inch is 25.30 mm. Prussian inch 26.15 mm. Austrian 26.34 mm. Parisian 27.07 mm.

In 1860 the International Congress of Ophthalmology proposed a system whereby lenses were to be numbered according to their refractive power. In 1872 the Ophthalmological Congress recommended a metric scale. This was adopted by the Ophthalmological Society in 1875. The unit was to be a lens of one meter (39.37 inch) focal length. This lens is known as one diopter. The words dioptry, diopter and dioptric are used in expressing the strength of a lens. It is abbreviated *D*. Thus 2.00 D.

72. The following table is not absolutely correct, but near enough so far as it concerns the refractionist.

Diopters	English inches	Centimeters	Millimeters
0.12	333.0	833.0	8333
0.50	80.0	200.0	2000
0.75	53.0	133.0	1333
1.00	40.0	100.0	1000
1.25	32.0	80.0	800
1.50	27.0	67.0	667
1.75	23.0	57.0	571
2.00	20.0	50.0	500
2.25	18.0	44.0	444
2.50	16.0	40.0	400
2.75	15.0	36.0	364
3.00	13.0	33.0	333
3.25	12.0	31.0	308
3.50	11.0	29.0	286
3.75	10.5	27.0	267
4.00	10.0	25.0	250
4.50	9.0	22.0	222
5.00	8.0	20.0	200
5.50	7.0	18.0	182
6.00	6.5	17.0	167
6.50	6.0	15.0	154
7.00	5.5	14.0	143
7.50	5.25	13.0	133

Dioptres	English inches	Centimeters	Millimeters
8.00	5.0	12.5	125
9.00	4.5	11.0	111
10.0	4.0	10.0	100
11.0	3.5	9.0	91
12.0	3.25	8.0	83
13.0	3.0	7.5	77
14.0	2.75	7.0	71
15.0	2.6	6.6	66.6
16.0	2.5	6.25	62.5
18.0	2.25	5.5	55.5
20.0	2.0	5.0	50.0

Optical Terms

73. **Opaque.**—Impervious to light. Bodies that transmit no light, such as pieces of wood and metal.

74. **Translucent.**—Semi-impervious to light. Bodies that transmit light, but so imperfectly as to obscure objects. Ground (frosted) glass, tissue and oiled paper are examples.

75. **Transparent.**—Pervious to light. Bodies that transmit light so perfectly that objects can be distinctly seen through them, such as clear glass and water.

76. **Ray of Light.**—This term applies to an extremely fine line of light. It may be looked upon as the direction of a light wave. The terms rays and waves are used interchangeably in this treatise.

77. **Beam of Light.**—A bundle of parallel rays. Such as emerge from the Emmetropic eye. A beam of light emerges from a convex lens or is reflected from a concave mirror when the luminous point is placed at the principal focus.

78. **Pencil of Light.**—A bundle or collection of converging or diverging rays. A converging pencil emerges from the Myopic eye. A diverging pencil from the Hypermetropic eye. If the object is placed within the principal focus of a convex lens or a concave mirror, or any distance from a convex mirror or concave lens, the resultant

cone of light will be divergent. If the object is placed outside the principal focus of a convex lens or concave mirror, the resultant cone will be convergent.

79. **Radiant Point.**—Any point upon an illuminated object may be spoken of as a radiant point, as from this point light is radiated.

80. **Absorption.**—The property to absorb light. Rays which impinge upon a substance and are not reflected have become absorbed.

81. **Refracted Ray.**—Ray which has entered obliquely a medium of different density and had its course changed.

82. **Incident Ray.**—Ray that has impinged upon a body.

83. **Angle of Incidence.**—Degree of the course which the ray takes before it impinges.

84. **Point of Incidence.**—Point where the ray impinges.

85. **Emergent Ray.**—Ray that has passed out of a medium.

86. **Normal to the Surface.**—At the point of incidence *N*, a line drawn perpendicular to the surface is normal. A straight line drawn from the center of the circle and through the circle is normal, perpendicular and at right angles to the circle. Fig. 60.

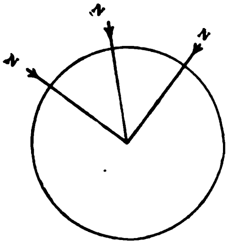


FIG. 60.

87. **Diffusion, Scattering of Rays.**—Imperfection in the curvature of a reflecting or refracting medium cause a scattering of rays. This results in a blurred distorted image as in irregular astigmatism.

88. **Images.**—A likeness of the object. An image results when the rays from an object are focused by a reflecting or refracting medium. The kind, size and location of an image depends upon the location of the object and the shape of the reflecting or refracting medium.

There are two kinds of images, real and virtual.

89. **Real Image,** termed Positive (+), becomes visible if intercepted by a screen as in a camera. Can be photographed. The inverted retinal image is positive.

90. Virtual Image.—Termed negative. (—) Imaginary. An optical illusion. Thus it cannot be photographed. Erect.

91. Aerial Image.—Image formed in the air. Observed with the ophthalmoscope.

Convex lenses form real images if the object is outside of the principal focus; and negative images if the object is within the principal focus. Concave lenses form negative images regardless of the location of the object.

Plane mirrors form negative images.

Concave mirrors and convex lenses form real images if the object is outside the principal focus, and virtual images if the object is within the principal focus.

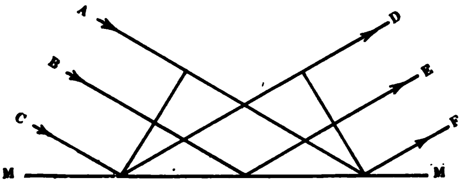


FIG. 61.

Convex mirrors and concave lenses, form negative images regardless of the location of the object.

92. Reflection of Light.—If an ether wave impinges upon a polished surface it glances off into the air. The wave is said to be reflected. Let *MM*, Fig. 61, represent a mirror surface. *A*, *B*, *C*, is a beam of light traveling toward the mirror. The wave *C*, strikes first and becomes *D*; *B*, becomes *E*; *A*, traveling a greater distance, becomes *F*.

The following law always holds good in reflection.

1. *The angle of incidence and reflection are always equal.*

2. *The incident and reflected ray are always in the same plane which is perpendicular to the reflecting surface.*

The incident ray *A*, Fig. 62, strikes the polished surface of B, and is reflected at the same angle, becoming the reflected ray *C*.

The perpendicular ray *D*, incident at *B*, is reflected back upon itself because it impinges normal to the surface.

From the diagram it will be seen that the angle of reflection *B*, *C*, *D*, equals the angle of incidence, *A*, *B*, *D*, and both angles lie in the same plane.

93. **A Plane is an Imaginary Flat Figure.**—For convenience it may be looked upon as having length and depth but not breadth. Fig. 63.

To demonstrate rule 2, stand a card upon a book so that the two will be at right angles, Fig. 63. The book represents the polished surface, the card represents the plane, *A*, *C*, *D*, *E*. *A*, represents the source of a ray; *B*, the point of incidence; *C*, the direction in which it is reflected. Thus both the reflected and incident ray lie in the same plane, which plane is perpendicular to the reflecting surface.

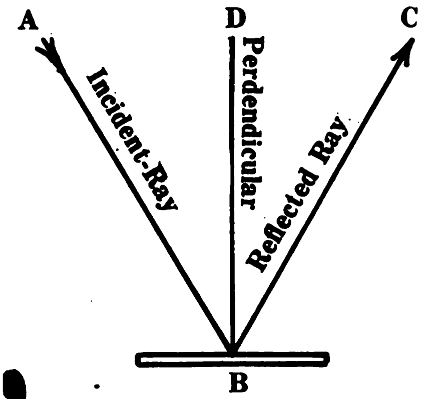


FIG. 62.

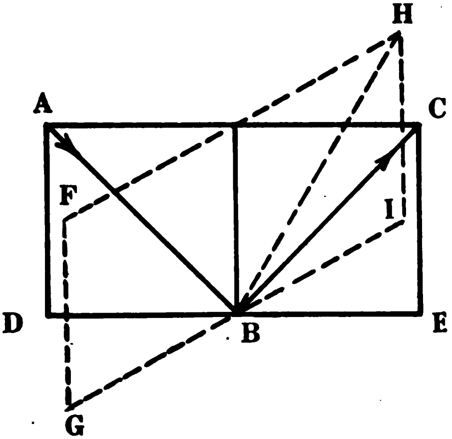


FIG. 63.

With *B*, as a center revolve the card, keeping it perpendicular to the surface. Each change in position however so slight represents another plane. Thus it will be seen that a complete revolution of the card would represent an infinite number of planes.

A ray from *A*, incident at *B*, would be reflected in the direction of

C, which is in the plane *A*, *D*, *C*, *E*. But if the reflected ray traveled in the direction of *H*, it would lie in the plane *F*, *G*, *H*, *I*, and the reflected and incident ray would not be in the same plane, which is never the case.

94. Deviation by Revolving Mirror.—The terms revolving, tilted and rotated are interchangeable.

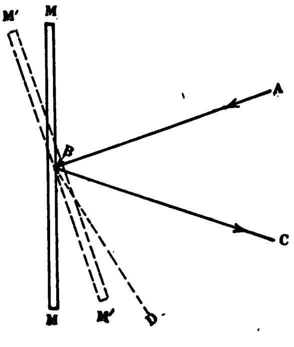


FIG. 64.

Axis of a mirror is an imaginary line, parallel with and extending through the middle of the surface.

Upon this imaginary line the mirror is revolved.

A retinoscope is revolved upon its handle. The axis extends through the handle. If a plane mirror is tilted upon its axis forward or backward, a ray of light incident upon it, will be reflected at an angle equal to twice that of the angle of deviation. Fig. 64.

The ray *A*, strikes the mirror *MM* at *B*, and is reflected in the direction of *C*. If the mirror is tilted in the plane *M'M*,' *A*, will be reflected in the direction of D. The ray is swept through the angle *CBD* which is twice the angle of *B' M' M*.'

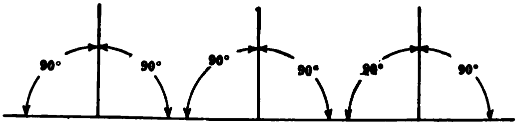


FIG. 65.

In reflecting Galvanometers this principle is utilized. The reflected ray is made to act as a long weightless indicator. Because of this principle the beginner in Retinoscopy experiences difficulty in keeping the reflected light upon the eye.

95. Plane Mirror.—*A plane mirror is a flat reflecting surface.* Therefore, every part lies in the same plane Fig. 65.

96. Images by a Plane Mirror.—Plane mirrors form virtual images which are of the size of the object. The distance of the image and object from the mirror are equal.

An eye at *C*, or *E*, Fig. 66, would see the image as though it were at F.

The image undergoes ***lateral inversion***, ***i.e.***, it apparently changes side for side.

97. Construction for Virtual Image Plane Mirror.—Fig. 66. *A*, represents the luminous point. A ray which strikes the mirror at *B*, is reflected in the direction of *C*. Another ray from *A*, strikes the mirror at *D*, and is reflected in the direction of *E*. *C*, and *E*, projected backward (see dotted lines) intersect at *F*, where a virtual image of the point *A*, is formed.

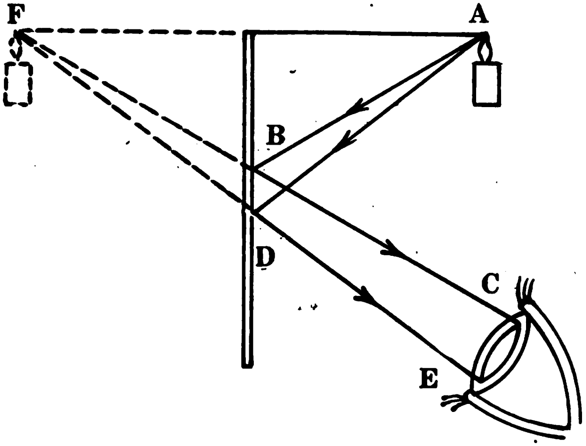


FIG. 66.

The Ray *B*, is from the upper part of the luminous point, and forms the upper part of the image of this point. The ray *D*, is from the lower part of the same point and forms the lower part of the image. Thus it can be seen why the image is upright.

Connect *A*, and *F*, (see dotted line), *F*, is as far back of the mirror as *A*, is in front of it.

To an eye placed at *C*, or *E*, the rays from *A*, would appear to come from *F*.

98. **Concave Spherical Mirror.**—A concave spherical mirror is a portion of a hollow sphere with the inner surface quick silvered. Fig. 67.

99. **Optical Center or Center of Curvature.**—Consider the mirror a complete circle. The optical center is situated at the center of the circle.

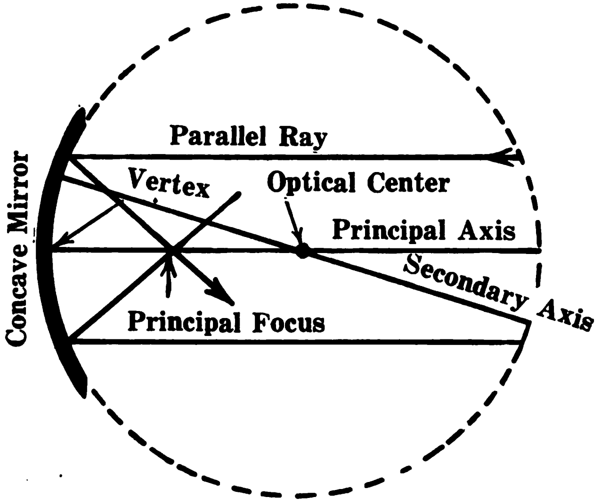


Fig. 67.

100. **Principal Axis.**—Is on a line drawn through the optical center, principal focus and pole. Principal axis is normal to the surface.

101. **Principal Focus.**—Midway between the optical center and mirror. Parallel rays after reflection are focused at this point.

102. **Pole or Vertex.**—Situated at the center of the mirror.

103. **Secondary Axis.**—Is on a line passing through the optical center, but not through the pole. It is normal to the surface.

104. **Radius.**—Applies to circles. One half of the diameter.

105. Radius of Curvature.—One half of the diameter of a circle that forms the mirrors surface.

106. Images Formed by a Concave Mirror.—When parallel rays strike a concave mirror they are reflected convergently and cross the principal axis at the principal focus, Fig. 67.

The size, kind, and location of the image depends upon the distance of the object from the mirror. Object placed as follows.

When between the principal focus and mirror the image is virtual, erect and enlarged, Fig. 69.

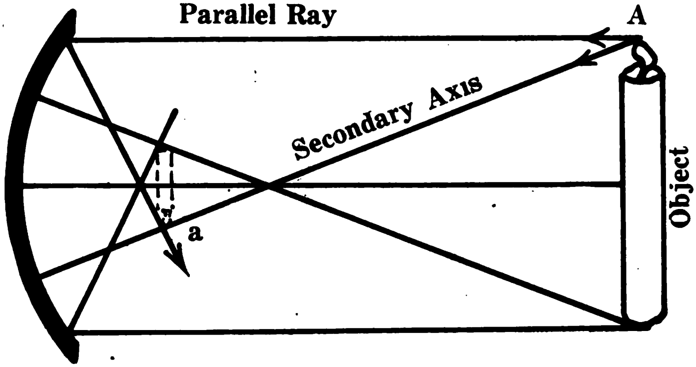


FIG. 68.

At the principal focus the image disappears because the rays after reflection become parallel. Between the center of curvature and principal focus the image is positive, inverted and enlarged, the rays being reflected convergently. Image formed beyond optical center.

At the center of curvature the image is positive, inverted and of the size of the object. Rays reflected convergently. Image formed at the center of curvature.

If beyond the optical center but at a finite distance Fig. 68, the image is real, inverted and smaller than the object. Rays reflected convergently. Image formed between principal focus and center of curvature, Fig. 68.

107. Construction for Images.—To locate the image of a point on an object it is necessary to trace but two rays from that point to where they intersect after reflection. Fig. 68, refraction Fig. 70, and projection Fig. 72. These points are conjugate. Draw one line parallel with the principal axis; the other line is traced along the secondary axis. The image of any point of the object may be constructed likewise.

108. Construction for Real Images, Concave Mirror.—Fig. 68. Start at the point *A*, on the object and draw a line parallel with the principal axis; let this line represent the parallel ray. From the point where it is incident on the mirror, continue it through and beyond the principal focus; let this line represent the parallel ray after reflection. From *A*, on the object draw another line through the optical center to the mirror; let this represent the secondary axis ray. At the point *a*, where these two lines intersect, an image of the point *A*, is formed.

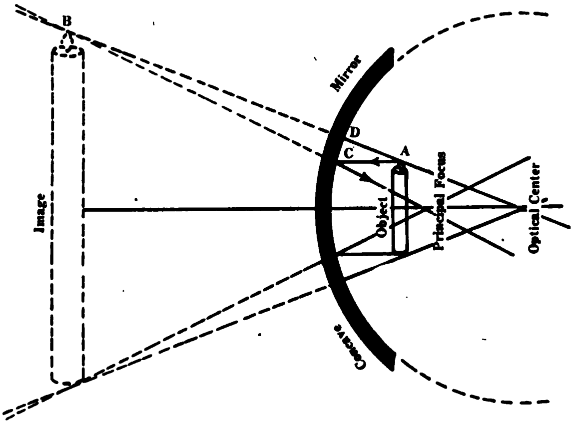


FIG. 69.

109. Construction for Virtual Image. Concave Mirror.—Fig. 69. Start at the point *A*, on the object and draw a line parallel with the principal axis to the mirror at *C*. Let this line represent the parallel ray. From *C*, continue this line through the principal focus; let this line represent the parallel ray after reflection. From *C*, project this line back of the mirror. Note the dotted line. Trace *a* line along the secondary axis through *A*, and to *D*. Project this line back of the mirror. Note the dotted line. At the point *B*, where these two dotted lines intersect, an imaginary image of the point *A*, is formed.

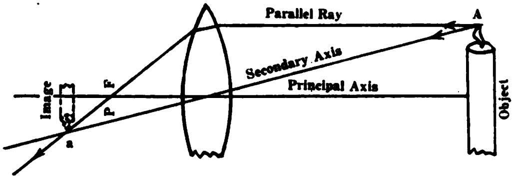


FIG. 70.

110. Convex Spherical Mirror.—A Convex Spherical Mirror is a portion of a hollow sphere, with the convex surface highly polished, Fig. 72.

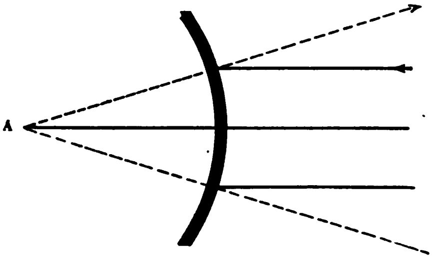


FIG. 71.

Parallel rays eminating from the object strike the convex mirror and are reflected divergent. See the parallel and reflected rays. Where the reflected rays if projected backward cut the secondary axial rays, a virtual erect image smaller than the object is formed. Convex mirrors form virtual images only.

111. Construction for Virtual Image. Convex Mirror.—Fig. 72. Start at the point *A*, on the object and draw a line parallel with the principal axis to the mirror; let this line represent the parallel ray. Continue this line through the principal focus; let this line represent the parallel ray after reflection. Trace a line along the secondary axis from *A*, to the optical center. Where the two dotted lines intersect at *a*, a virtual image of the point *A*, is formed.

112. Magnifying Power Concave Mirror.—Fig. 73. *A*, *B*, is the magnified image of *C*, *D*, the object which is located between the principal focus *F*, and the center of curvature *O*. The ratio of *A*, *B*, to *C*, *D*, stands for the linear magnifying power of the mirror.

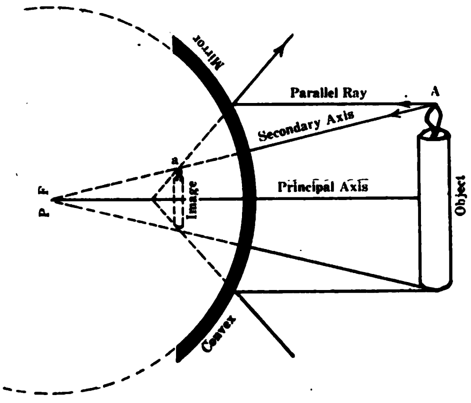


FIG. 72.

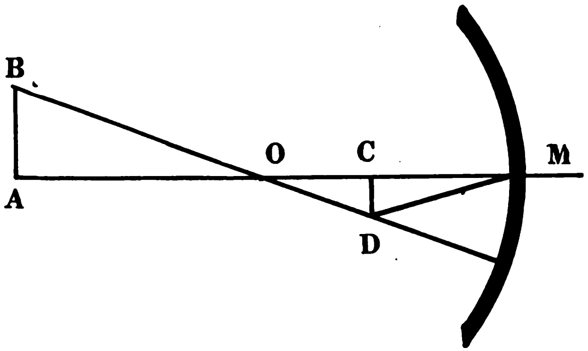


FIG. 73.

The triangles *A*, *B*, *O*, and *O*, *C*, *D*, are similar. Then

$$\frac{\text{AB}}{\text{CD}} = \frac{\text{AO}}{\text{OC}} = \frac{\text{Distance of image from O}}{\text{Distance of object from O}}$$

Also the triangles *C*, *M*, *D*, and *A*, *M*, *B*, are similar. Thus

$$\frac{AB}{CD} = \frac{AM}{CM} = \frac{\text{Distance of image from mirror.}}{\text{Distance of object from mirror}}$$

If a plus spherical lens is held between the sun and a piece of paper, and the light is brought to a focus, the result is a small round area of illumination upon the paper. If a plus cylinder is used the result is a narrow band.

It is interesting to focus the sun's rays through various combinations of spheres and cylinders. Parallel rays focus at a shorter distance from a lens than those divergent. Rays whether parallel, convergent or divergent, suffer the same amount of refraction, it then follows that: The closer an object is to a lens the farther back the focus or image of the object occurs. Because the closer the object the more divergent are the waves that impinge upon the lens.

113. Refraction of Ether Waves.—Careful experiments have demonstrated that light passes through air, water glass, alcohol, etc. at a less speed than through a vacuum. This property of retarding light is referred to as optical density. The optical density usually increases with the mass density, but not always. To illustrate, Huyghens wave theory, we will assume the wave *A*, *A*, Fig. 74, traveling from a point in the rarer medium air through the denser medium glass. The wave front is represented by *B*, *C*. It will be observed that both edges *B*, *C*, enter the glass simultaneously. Both edges are retarded and while the speed of the entire wave slows down, yet its direction remains unchanged, and it passes out following the original direction, and also regains its former speed. The wave *A*, *E*, Fig. 74, impinges at an angle with the perpendicular *P*, *P*, and the edge *C* enters first and is retarded; the edge *B*, still in the air and traveling faster, swings the wave front until *B*, also enters, which is toward the perpendicular *P*. Both edges then travel apace, while in the glass. *C*, emerges first and *B*, still retarded, it can be seen that *C*, will forge ahead and swing the wave away from the perpendicular *P*, *P*, until *B*, passes out. Both edges are now in the same medium and the wave will continue in a straight course. An object at *E* appears to be at *D*, which is in line with the course of the wave as it enters the eye at *A*.

It will be observed that the change in direction takes place at the surface of the substance. This change in direction is known as *refraction.* Thus we obtain the following laws:

1. *A wave passing from a rare to a denser medium is refracted toward the perpendicular.*

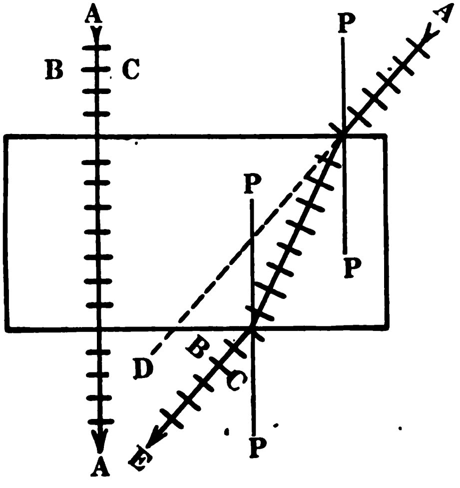


FIG. 74.

2. *A wave passing obliquely from a denser to a rarer medium is refracted away from the perpendicular.*

114. Refraction by a Prism.—Fig. 75. The waves *A*, *A*, *A*, pass through the prism and remain parallel. As *A*, *B*, do not cross, no image is formed. Fig. 75. A wave from *A*, after refraction enters the eye at *B*. *B*, is projected outward in the direction of *C*. *A*, appears to be at *C*. This displacement is spoken of, as the prismatic effect.

115. Refraction by Curved Surface (Lenses).—Remember that the refracting power of a medium depends upon its curvature and index of refraction.

The sharper the curve, or the higher the index of refraction, the greater is the refractive power. Because the sharper the curve, the greater is the length of time the edge of the wave in the rarer medium has to travel before entering the denser medium. Also, the higher the index of refraction, the more retarded is the edge of the wave which has entered the denser medium. Figs. 76, 77.

A wave passing through the principal axis of a curved surface, enters and emerges simultaneously. But the wave *C*, Figs. 76, 77, travels parallel with the principal axis but outside. One edge enters first and is retarded; the other edge still in the air and traveling faster swings toward the edge impinging first. On emerging, the edge first retarded passes out first. Thus parallel waves are bent toward the thicker portion of a lens whether plus or minus. The action of spherical lenses, on light, may be briefly summarized:

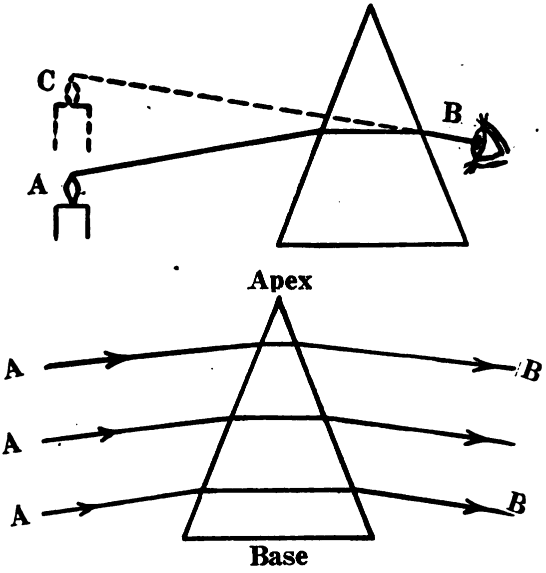


FIG. 75.

Parallel waves of light are refracted toward the thicker portion of a lens.

116. Refraction by a Convex Surface.—The wave *C*, Fig. 76, impinges upon the bi-convex lens and edge *B*, enters first and is retarded. *A*, still in the air and traveling faster, swings the wave. This change of direction occurs until both edges have entered the glass, when the wave travels in a straight line. *A*, emerges first and traveling faster than *B*, again changes the direction. When *B* emerges the wave travels in a straight path. Assuming that the wave when incident was parallel with the principal axis, it would cross this line at *B*, *B*, is the principal focus.

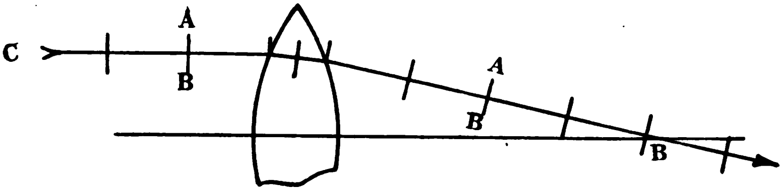


FIG. 76.

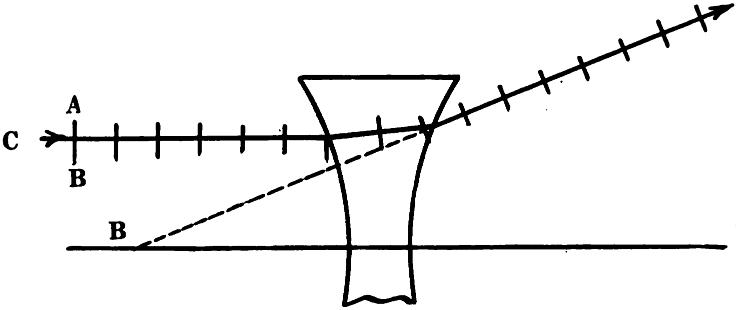


FIG. 77.

117. Refraction by a Concave Surface.—The wave *C*, Fig. 77, impinges upon the bi-concave lens and the edge *A*, enters first and is retarded. *B*, still in the rarer medium air, and traveling faster swings the wave until it has also entered. *B*, emerges first and swings the wave still more until *A*, is also in the air. The wave then travels in the direction of *C*. *C*, projected backward locates the point *B*, which is the principal focus of the lens.

It can be seen that refraction takes place at each surface. Also that the sharper the spherical curve the greater will be the length of time that the edge in the air has to swing the wave.

118. Action of a Convex Cylinder on Parallel Rays.—Parallel rays that have undergone refraction by a convex cylindrical lens are not converged to a single point as by a convex spherical lens. Instead they are focused in a line known as the principal focus, Fig. 78. This lens has no refractive power through the principal axis. Its maximum strength lies in the meridian at right angles to the principal axis, and gradually diminishes to zero. Suppose Fig. 78 is a +5.00 D lens. At 45° which of course is halfway between the meridians of full strength and zero, the refractive power is +2.50D.

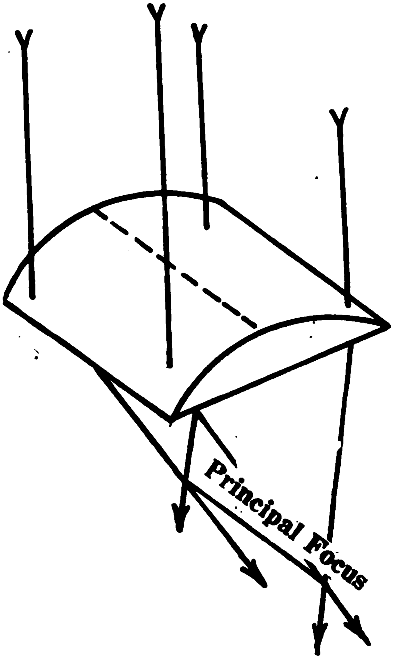


FIG. 78.

119. Action of a Concave Cylinder on Parallel Rays.—Fig. 79. The action of a concave cylinder is opposite to that of a convex cylinder, *i.e.*, the rays after refraction diverge. Otherwise the foregoing description is applicable.

Cylindrical lenses are used in correcting astigmatism.

120. Conjugate Foci; Convex Lens.—If an object is placed at twice the focal length of a convex lens, which is at *A*, Fig. 80, rays from it would be focused at the same distance on the other side at *B*. Or rays from *B*, would focus at *A*. *A*, and *B*, are conjugate to each other.

If an object was placed within the principal focus, say at *D*, rays from it would strike the lens divergent and emerge divergent, but less so than before striking it. If the emergent rays were continued backward, they would meet at a point outside the principal focus. Therefore *E*, is a virtual focus of *D*, and it is referred to as negative.

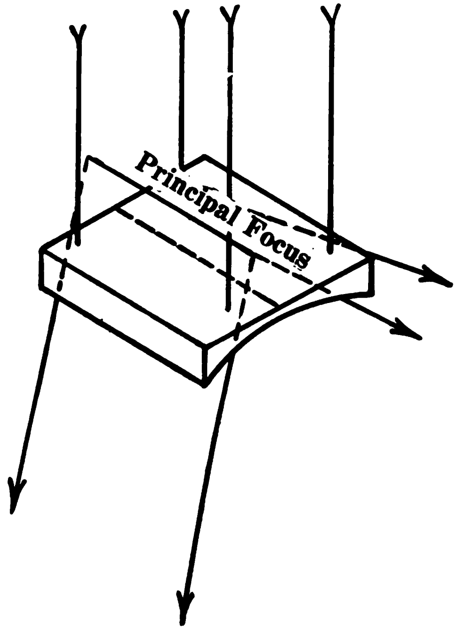


Fig. 79.

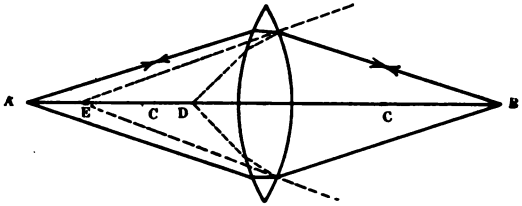


Fig. 80.

121. Optical Center or Nodal Point of a Lens.—*The optical center or nodal point of a lens is that point on the optic axis through which the direction of principal and secondary axial rays remain, unchanged.*

This point in bi-convex and bi-concave lenses is midway between the two surfaces. In meniscus lenses it may be outside of the lens.

122. Geometrical Center.—*A point equally distant from all edges.* A secondary axis is a line drawn through the optical center of a lens or mirror, but not in line with the principal axis. In passing through the secondary axis the course of a ray is changed but not its direction. Fig. 81.

123. Construction for the Course of a Secondary Axis Ray.—Draw the lens and centers of curvature, Fig. 81. The ray *A*, incident at *B*, passes through the optical center *O, C*, and emerging at *C*, continues in the direction of *D*. Project *C, D*, backward (see dotted line), to where this line crosses the principal axis at *E*, *a nodal principal point is located.* Project the ray *A, B*, (see dotted line) to *F*. Where it crosses the principal axis at *G*, the other nodal point is located.

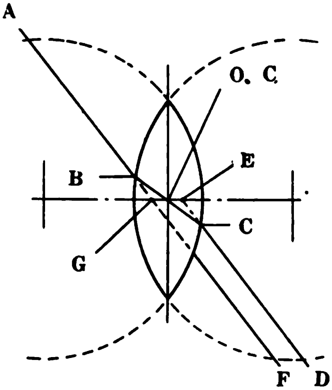


FIG. 81.

From the diagram it can be seen that the course but not the direction of the ray has been changed. In thin lenses the nodal points and optical center are so close together that they may be considered as one.

124. Images by a Convex Lens.—Kind, size and location of, with the object placed as follows:

Between the lens and its principal focus the image is virtual, erect and enlarged. Fig. 82.

At the principal focus of the lens no image is formed because the rays after being refracted emerge parallel.

If beyond the principal focus, but at less than twice the focal length, the image is real, inverted, magnified, and is formed beyond the principal focus.

If at twice the focal distance the image is real, inverted, of the size of the object, and located at twice the focal length.

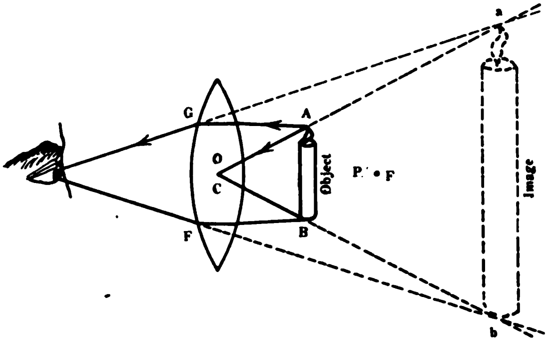


FIG. 82.

If at infinity or more than twice the focal length the image is real, inverted, smaller than the object, and located outside the principal focus. Fig. 83.

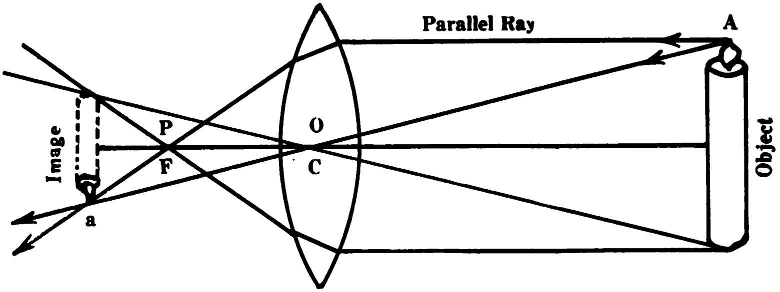


FIG. 83.

125. Construction for a Real Image; Convex Lens.—Starting from the point *A*, Fig. 83, on the object draw a line parallel with the principal axis; let this line represent a parallel ray. After refraction continue this line through the principal focus.

From *A*, trace a line along the secondary axis through the optical center. At *a*, where these two lines intersect, an image of the point *A*, is formed.

126. Virtual Image; Convex Lens.—Remember that if the object is situated within the principal focus, Fig. 82, the rays will impinge upon the lens so divergent that they will emerge divergent after refraction, but less so than before entering it. The image is judged to be in line with the course of the rays entering the eye. Thus the lens acts as a magnifier. This image is virtual, erect, enlarged and is apparently formed on the same side of the lens as the object.

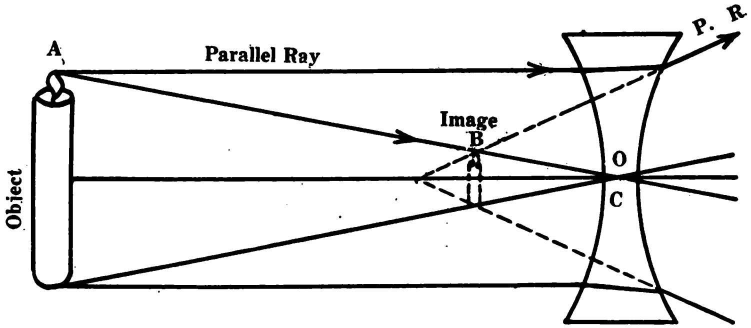


FIG. 84.

127. Construction for a Virtual Image; Convex Lens.—Starting from the point *A*, Fig. 82, on the object draw a line parallel with the principal axis. Let this line represent *a*, parallel ray. From the point where it emerges from the lens continue it to the eye. From *A*, draw another line to the optical center.

Project these two lines, and where they cross at *a*, an image of the point *A*, is apparently located. Observe that *a*, is in line with the course of the ray which enters the eye.

128. Virtual Image; Concave Lens.—Concave lenses form virtual images only, which images are erect and smaller than the object. As the object is caused to recede, the image decreases in size, Fig. 84.

A parallel ray from *A*, passes through the lens and is refracted divergently. Another ray passes through the optical center. Where the refracted ray if projected backward crosses the secondary axial ray, which point is at *B*, a virtual image of *A*, is formed.

129. Construction for Virtual Image; Concave Lens.—Starting from the point *A*, Fig. 84 on the object draw a line parallel with the principal axis; let this line represent the parallel ray. Draw another line along the secondary axis. Project the refracted ray backward, and where it cuts the secondary axial ray at *B*, a virtual image of *A* is located.

130. Formation of a Retinal Image.—From what has been said on the formation of images by lenses and mirrors, it can be seen that the most perfect image as formed by the dioptric media of the eye, also occurs where the refracted ray crosses the secondary axial ray.

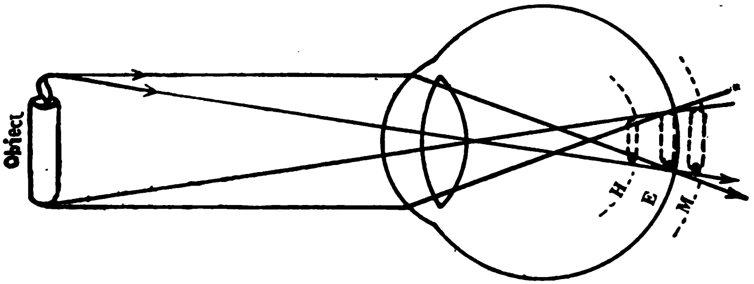


FIG. 85.

Fig. 85. In Emmetropia the retina is located at the point of intersection the length of the eye ball being normal.

In Hypermetropia the point of intersection is back of the Retina because the eye ball is abnormally short. The image is imperfectly defined, because the rays have not come to a focus. A plus lens of the exact dioptric strength would cause the parallel rays to so converge that they would cross the secondary axial ray at the retina.

In Myopia, the point of intersection is in front of the retina, because the eye ball is abnormally long. The image is imperfectly defined, because the rays have focused in the vitreous and are diverging when they impinge upon the retina. A minus lens of the exact dioptric strength would cause the parallel rays to so diverge before entering the eye that they would cross the secondary axis ray at the retina.

From the foregoing it is clear that Hypermetropia and Myopia can be corrected.

The retinal image is positive and inverted.

The visual angle is formed by the secondary axes where they cross at the nodal point. The size of the retinal image depends upon the size of the visual angle. By referring to the sketch Fig. 85, it can be seen that the retinal image is smaller in Hypermetropia than in Emmetropia, and smaller in Emmetropia than in Myopia. The size of the visual angle depends upon the size and distance of the object. In judging the size and distance of an object the size of the visual angle leads to a mental calculation. The size and distinctness of surrounding objects are also considered.

131. The Eye as an Optical Apparatus.—The eye in structure may be looked upon as a camera of miniature dimensions. The refractive media of the eye corresponds with the lens of the camera; the iris to the iris diaphram; the eye ball to the darkened box; and the retina to the ground glass screen. They differ however in the focalization of light. In the eye to secure a well defined image upon the retina, the refractive power of the crystalline lens must be altered as the distance of objects vary. With the camera it is necessary to move the screen back or forth.

132. Lens Formula.—We will assume that the lens is very thin and of small curvature. Let u, equal distance between object and lens. Let v, equal distance of image. Let f, equal the focal length. Formula reads:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

The object and image if real are on opposite sides of the lens, *f*, is constant, *v*, diminishes as *u*, increases. The object and image move in the same direction.

If the image is virtual, e is negative.

133. Tracing a Ray Through a Convex Lens.—Draw the lens centers of curvature *CC′* and principal axis Fig. 86.

Draw the incident ray *A*, *B*, to be traced through the lens; *B*, is the point of incidence. Draw the normal *C′B*.

The index of refraction is $3/2$. Taking *B*, as a center point off three equi-distant spaces on the dotted line *B*, *C′*, the normal, and using the two extreme points draw arcs through them, *Q*, *D*, and *E*, *F*. Where the incident ray *A*, *B* cuts the arc *E*, *F*, which is at *G*, draw a line parallel to the normal *C′*. From where this line cuts the arc *Q*, *D*, which point is at *H*, draw a line to the point of incidence *B*, of the ray *A*. The continuation of this line through the lens represents the course of the ray while in the lens. Continue it for some distance, as it is used in constructing for the emergent ray, as on emerging from the lens the ray will again be refracted.

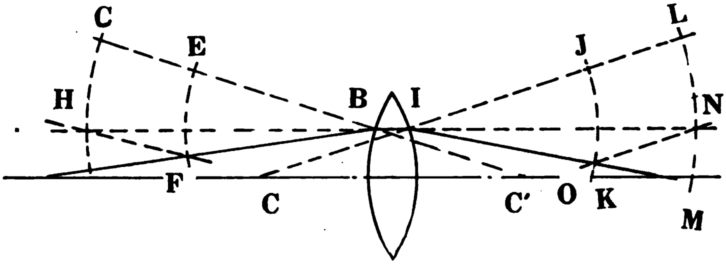


Fig. 86.

Draw the normal *C*, *I;* using *I*, as a center draw the arc *J*, *K*, and *L*, *M*. Where *L*, *M*, intersects the line *H*, *B*, which point is at *N*, draw a line parallel to the normal *C*, *I*. The refracted ray will pass through the point where the line *I*, *N*, intersects the arc *J*, *K*, which is at *O*, and travel in the direction of *P*.

134. Tracing a Ray through a Concave Lens.—Fig. 87. From *B* swing the arcs *C*, *D*, and *E*, *F*. Where the ray *A* crosses the arc

E, F, which is at *G,* draw a line parallel with the normal *C, B,* and project it through *H.* The line *H, B* will indicate the course of the ray through the lens. From *D,* swing the arcs *J, K,* and *L, M.* From where *H, B,* cuts the arc *L, M,* which point is at *N,* draw a line parallel with the normal *C′,D.* This line becomes *N, T.* The emergent ray will pass through the point *T.*

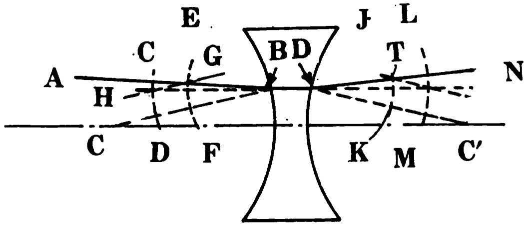


FIG. 87.

135. To Determine the Principal Focus.—From *B,* draw the arcs *C, D,* and *E, F,* Fig. 88. Where the incident ray *A,* crosses the arc *E, F,* at *G,* draw a line parallel with the normal. Where this line crosses the arc *C, D,* at *H,* draw a line through *B,* and beyond the lens. This is the course the ray will take while in the lens. With *I,* the point of emergence as a center, draw the arcs *J, K,* and *L, C′.* From *N,* draw a line parallel to the normal. Where this line cuts the arc *J, K,* at *O,* the emergent ray will pass through and meet the principal axis at *C′*, which is the principal focus.

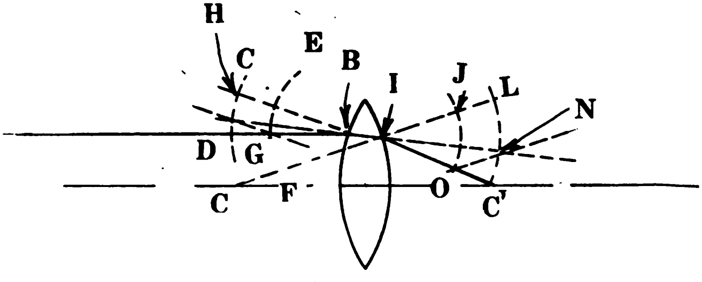


FIG. 88.

136. Tracing a Ray through Plane Glass.—Fig. 89. Draw the incident ray *A*, to *B*, the point of incidence. Through *B*, draw the normal *P*. With *B*, as a center draw the arcs *C*, *D*, and *E*, *F*. From where the ray *A*, cuts the smaller arc *E*, *F*, at *G*, draw a line parallel to the normal *P*. From where this line cuts the larger arc *C*, *D*, at *H*, draw a line through *B*, and the glass. This line will be the course the ray will take while in the glass. With *I*, the point of emergence, as a center, draw the arcs *J*, *K*, and *L*, *M*. Project the line *B*, *I*, and from where it cuts the larger arc, *L*, *M*, which point is at *N*, draw a line parallel with the normal *P*. At the point where this line cuts the smaller arc, *J*, *K*, at *O*, the emergent ray from *I*, will pass through.

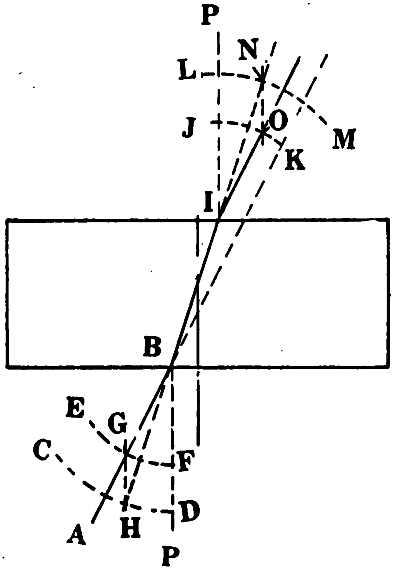


Fig. 89.

137. Tracing a Ray Through a Prism.—Fig. 90. Draw the incident ray *A*, to the point of incidence *B*. Through *B*, draw the normal *P*. With *B*, as a center draw the arcs, *C*, *D*, and *E*, *F*. From where the ray *A*, cuts the smaller arc *E*, *F*, at *G*, draw a line parallel to the normal *P*. From where this line cuts the larger arc *C*, *D*, at *H*, draw a line through *B*, and the prism. This line will be the course of the ray while in the glass. With *I*, the point of emergence, as a center, draw the arcs *J*, *K*, and *Z*, *M*. Project the line *B*, *I*, and from where it cuts the larger arc *Z*, *M*, which point is at *N*, draw a line parallel with the normal *P*. At the point where this line cuts the smaller arc *J*, *K*, at *O*, the emergent ray from *I*, will pass through traveling in the direction of *Q*.

138. Figuring Lenses Wave Theory.—From the wave theory we have learned that light is the result of a transverse disturbance in the ether. So far we have referred to a single wave front only. We will now consider beams and pencils of light. In Fig. 91, *S*, represents a luminous or radiant point. From *S*, waves are propagated and the wave front indicated by the various circles. It can be seen that the curve flattens as the wave travels outward. At a distance of twenty feet the front is quite flat, so much so that it is considered a plane surface. If the pupil of an eye is taken at ⅛″ in diameter, the small portion of the front that enters the pupil may be spoken of as plane without serious error. The lines *A*, *B*, diverge so little that they are treated as parallel rays.

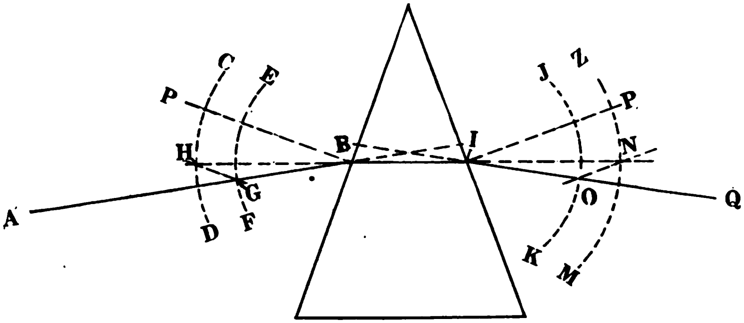


FIG. 90.

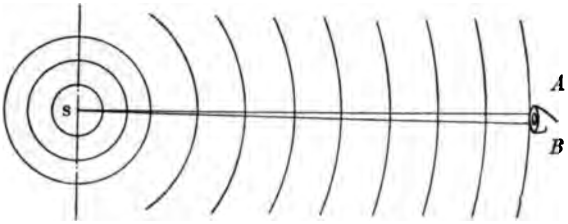


FIG. 91.

Strictly speaking there are no parallel rays in nature. Even those propagated by the sun at a distance of 93 million miles are slightly divergent.

Diverging rays are designated minus (—). Rays that have been rendered convergent by a plus lens or convave mirror are plus (+) rays. The distance traveled by a ray is expressed in diopters. These figures correspond with the table of focal lengths of lenses, *e.g.* Fig. 91, the ray which has traveled 20 inches is expressed 2.00 D. etc.

139. To Find the Focal Length of a Plus Lens.—Fig. 92. We will assume that the wave front *B*, *C*, *D* has traveled a great distance and that *B*, *C*, *D* is a straight line. The lens is +4.00 D. and has two surfaces each +2.00 D. *C*, enters the denser medium glass and is retarded. *B*, and *D*, in the rarer medium air travel faster than *C*. When the three parts are inside it can be seen that the +2.00 D. curve has changed the plane front to a +2.00 D. curved front. *B*, and *D*, after traversing the glass pass out into the air and regaining their original speed they forge ahead of retarded *C*. Thus the second surface has given *B*, *C*, *D* an additional +2.00 D. curve. The wave front is now expressed +4.00 D. Thus *B*, *C*, *D*, cross the principal axis at *F*, the focus of a +4.00 D. lens, which is 10 inches.

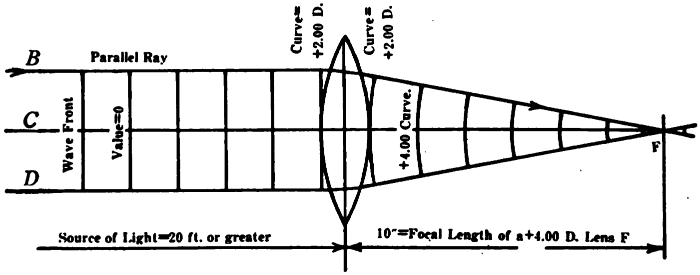


FIG. 92.

140. To Find the Focal Length of a Concave Lens.—Fig. 93. The wave front *B*, *C*, *D* is plane. *B*, and *D*, enter the glass and are retarded. *C*, in the air traveling faster bulges the wave front inward. The first surface of the minus 4.00 D. lens is —2.00 D.

Hence, the wave front traveling in the glass is −2.00 D. Upon emerging *C*, passes out first and bulges the wave front an additional −2.00 D. The lines *B*, *D*, projected backward cross the principal axis at *F*. *F*, is the principal focus and is expressed minus (−).

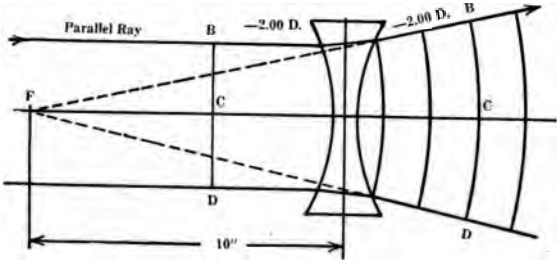


FIG. 93.

From the foregoing it can be seen that the action of a concave lens upon parallel rays is opposite to that of a convex lens.

141. Reflection by a Concave Mirror.—Fig. 94. The pencil of light from *L*, has a curve of 1.00 D. After reflection it has a 2.00 D. curve and comes to a focus at 20 inches.

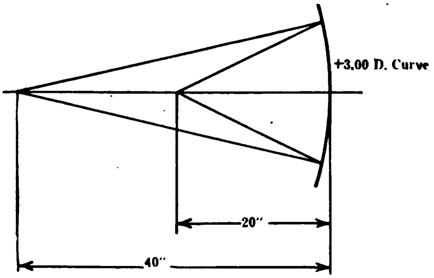
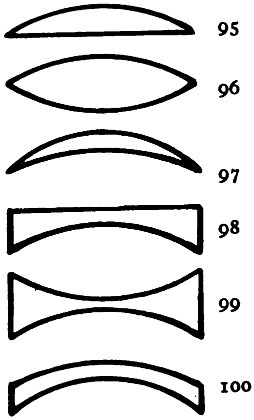


FIG. 94.

142. Lenses, Kinds and Forms of Both, Spherical and Cylindrical.

—Lenses are ground in two forms—Convex and Concave.

Convex lenses, Fig. 95, 96, 97, are thickest at the center. They are termed plus, positive and are given the sign +. They are also referred to as magnifying and burning glasses. Their use is to correct hypermetropia and aid the accommodation in presbyopia.



FIGS. 95 TO 100.

Concave lenses are thickest at the edge. They are termed minus, negative and are given the (−) sign. They are also referred to as minifying glasses. They correct a myopic condition.

Convex and concave lenses are ground in both the spherical and cylindrical form. A spherical lens is of the same strength in all meridians, and corrects axial ametropia. A cylindrical lens is the segment of a cylinder, and does not possess the same strength in all meridians. It is used to correct curvature ametropia.

Plus and minus lenses are ground in the following forms:

Plano-convex has a plane and convex surface, Fig. 95.

Bi-convex has two convex surfaces, Fig. 96.

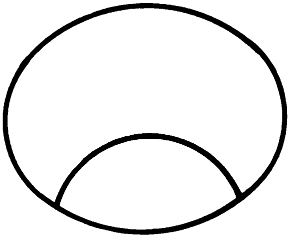


FIG. 101.

Periscopic-convex, Fig. 97. The radius of the convex surface is shorter than the radius of the concave surface.

Plano-concave, Fig. 98, has a plane and concave surface.

Bi-concave, Fig. 99, has two concave surfaces.

Periscopic-concave, Fig. 100. The radius of the concave surface is shorter than the radius of the convex surface.

Bi-focal lenses, Fig. 101, are of double focus. The upper part is for distant vision; the lower part is for near-by vision. Bi-focals are used in correcting presbyopia, and in cases where the accommodation is so reduced that it is inadequate. Bi-focals should be made light as possible and aberration at the edge of the segment minimized. There are many forms of the bi-focal.

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Periscopic Lens.—Fig. 102. The periscopic form of lens is preferable to the bi-convex and bi-concave, because in looking through the edge, aberration and the prismatic effect is less. Also the field of vision is enlarged.

Toric Lens.—A toric lens is one in which the strength of the two principal meridians are ground on the same side of the lens. On the other side is ground as desired a plano, convex or concave sphere. The deep periscopic form is obtained if strength is added to the toric side, and the same strength but of opposite sign to the other side. Sphericals in toric form are expensive.

FIG. 102.

143. Base Curve.—Meridian of least refractive power on the toric surface.

144. Cuts Showing Approx. the Different Size of Eye.—Figs. Jumbo to *X* inclusive represent the different sizes of eye. Reproduced from F. A. Hardy & Co.'s catalog.

Eye	Inserted	Frameless	Short oval frameless
Jumbo	35.4 × 45.4 mm.	38.0 × 46.0 mm.	39.5 × 44.5 mm.
0000	35.5 × 44.5 mm.	36.0 × 44.0 mm.	37.5 × 42.5 mm.
000	33.5 × 42.5 mm.	33.5 × 42.5 mm.	35.0 × 41.0 mm.
000	32.0 × 41.0 mm.	32.0 × 41.0 mm.	33.5 × 39.5 mm.
00	30.7 × 39.7 mm.	31.0 × 40.0 mm.	32.5 × 38.5 mm.
0	28.8 × 37.8 mm.	29.5 × 38.5 mm.	31.0 × 37.0 mm.
1	27.5 × 36.5 mm.	28.0 × 37.0 mm.	29.5 × 35.5 mm.
2	26.0 × 35.0 mm.	27.0 × 36.0 mm.	
A	24.7 × 38.5 mm.		
B	22.5 × 39.5 mm.		
C	21.0 × 36.0 mm.		
D	20.7 × 35.0 mm.		
F	half eye any size		
X	38 round.		
4			
0–5.0			
00–5.0			
000–5.0			
0000–5.0			
1–5.0			

145. To Determine the Strength of a Plus or Minus Lens.—If it is desired to know the strength of a lens from which the number tag has been lost, proceed as follows: First, ascertain if it is a plus or minus lens. Then take from the trial case a lens of opposite sign, and known strength. Place the two lenses together and move them before an object. If the object apparently moves, try another lens from the trial case until there is no apparent movement of the object. One lens is said to neutralize or overcome the prismatic effect of the other. The lens being tested corresponds in strength to that of the lens of known sign.

Example.—If a minus lens, say 1.00 D. is placed over a plus 1.00 D. there will be no apparent movement of the object.

In testing lenses more accurate results are obtained if the following suggestions are borne in mind. Plano convex and plano concave lenses make excellent test lenses. The two best fitting surfaces should be placed together. The optical centers of both lenses should be on a line. Both lenses should touch and be held parallel to each other.

146. To Determine Whether a Lens is Plus or Minus.—Hold the lens close to the eye and look through it at some distant object; then move the lens from side to side and up and down, in fact move it in all of the different meridians. If the object apparently moves with the lens, it is minus. If the object apparently moves in an opposite direction the lens is plus. If all meridians produce movement of the same direction and rate, it is a spherical lens. The greater the refractive power the greater is the prismatic effect of a lens. It then follows the more rapid also is the apparent movement of the object.

If one meridian produces no movement, and all others do, the lens is a simple cylinder. The meridian which produces no movement is in line with the cylindrical axis.

If the movement in the two principal meridians is in an opposite direction, the lens is a plus or minus compound.

If the movement in both meridians is in the same direction, but *differ in rate, it is a plus* or minus compound.

147. To Find the Optical Center.—Hold the edge of a lens over a straight line. If the lens is convex, the line will appear displaced outward. If the lens is concave, the line will appear displaced inward. Move the lens outward, looking through the center; when the line appears continuous, make a mark upon each edge to correspond. Draw through the first line another at right angles and proceed as before. At the point where these two lines intersect the optical center is located.

The axis of a cylinder can be determined by looking through the lens at a straight edge, Fig. 103, and rotating the lens until the line appears continuous, Fig. 103. The straight line will be in line with the axis.

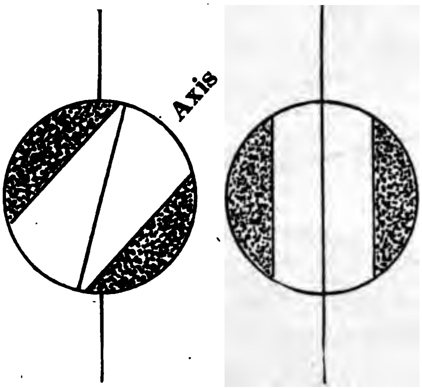


FIG. 103.

148. Magnification.—To understand magnification, we must know more about the visual angle, Fig. 104. The increased size of a near-by object over one at a distance, is due to the larger retinal image, or in other words, the greater angle at which the rays enter the eye. It is a fact that an object appears to be almost in line with the course of the rays as they enter the eye. Or perhaps more strictly speaking rays that have focused upon the retina and are then projected outward. Points on the object appear to be in line with these projected rays.

In Fig. 104 we will let *A, B,* represent a remote object, and the size of the retinal image is determined by the angle *C, D.* Let *E, F,* be a smaller object but placed closer to the eye. It can be seen that it will form the image *G, H,* which is of a larger size.

In estimating the magnifying power of a lens 10 inches 25 centimeters is taken to be the shortest distance of distinct vision.

149. A Simple Microscope.—The ratio of the apparent diameter of the image to the diameter of the object, placed at a distance of 10 inches equals the approx. magnifying power.

Referring to Fig. 82, the ray *A,* after refraction enters the eye. Projected from the retina outward it cuts the line *A, C.* At this point a virtual image of *A,* is formed. It can be seen that the visual angle *E, F, G,* is much larger than the angle included between the lines from *E,* to *A, B.* In just a word the lens acts as a magnifier because it increases the size of the visual angle. An object at a distance of 1000 feet appears to be 1/10 the size of the same object at one hundred feet.

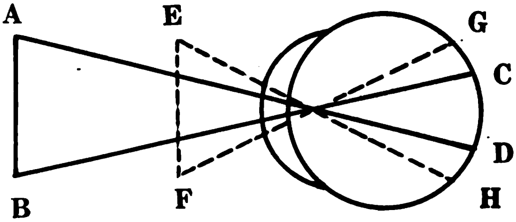


FIG. 104.

The magnifying power of a lens can be found from the formula 25/*f* which means 25 divided by the focal length in centimeters, *e.g.*, focal length equals 2.5 cm. The magnification would be 10 diameters, the object placed at the focal length.

150. Compound Microscope.—Fig. 105 represents the arrangement. *O,* is the objective lens and of short focus. *E,* is the eye piece and of longer focus. The object *A, B,* is placed slightly outside of the focus of *O.* An enlarged real image is formed at *C, D.* *C, D,* lies within the focus of *E*, and appears to be at *F*, *G*. *F*, *G*, is virtual and enlarged.

151. Magnification, Compound Microscope.—The magnifying power may be expressed as the product of the power of both lenses, *e.g.*, objective magnifies 25 times. Eye piece magnifies 10 times. 25 × 10 = 250 times.

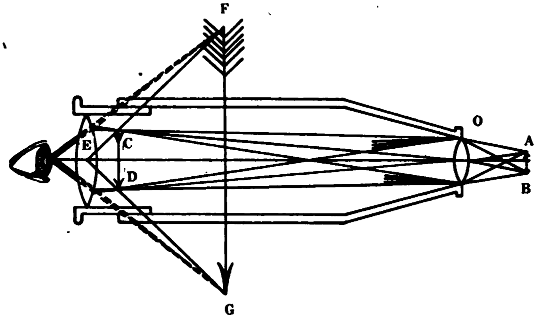


FIG. 105.

152. Astronomical Telescope.—Fig. 106. We will assume that the object *A*, *B*, is at a great distance. The image *C*, *D*, is real, inverted and diminished. *C*, *D*, is viewed through the eye piece *E*, and appears to be at *F*, *G*. This form of instrument is known as a refracting telescope.

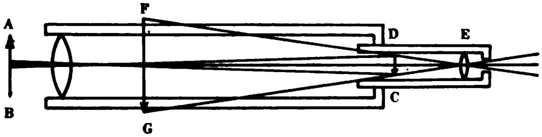


FIG. 106.

153. Magnification Telescope.—The magnifying power is found by dividing the focal length of the objective by the focal length of the eye piece.

154. The Opera Glass.—Fig. 107. The convex objective *O* would produce a real image at *A*, *B*, but the rays are intercepted by the eye piece *E*, and caused to diverge to *C*, *D*, where an erect magnified image appears.

155. The Diaphragm and Size of Aperture.—A diaphragm with adjustable apertures is used in the photographic camera. Its purpose is to regulate the amount of light so as to obtain the correct exposure. The rapidity of a lens is reckoned from the size of the aperture. The diameter is expressed as a fraction of the focal length, *e.g.*, (f 8) indicates that the diameter of the opening is ⅛ of the focal length of the lens.

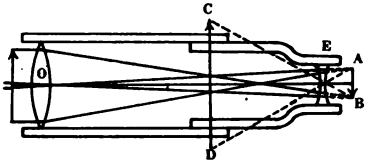


FIG. 107.

The following table was recommended by the Photographic Society of Great Britain.

f	f	f	f	f	f	f	f	f Relative exposure ratio
4	5.6	8	11.3	16	22.6	32	45.3	64
1	2.0	4	8.0	16	32.0	64	128.0	256

CHAPTER III

VISION

156. When ether waves ranging within certain limits of wave length impinge upon the retina, nerve impulses are created. These impulses are conveyed to the cortical centers of the brain. Vision is the result.

We have been taught that the refractive media of an eye forms an inverted image of objects upon the retina. The visual act consists of transforming this image into a mental picture.

In order to see distinctly we must have a perfectly defined image upon each retina. To obtain a sharply defined focus the eyes must accommodate. Also the recti muscles must adjust the visual axes so that the images will be formed upon corresponding points of the two retinas.

There are two theories as to how vision is produced.

157. Mechanical Theory of Vision.—It is assumed that the impact of the ether waves are the source of nerve impulses. These waves acting upon the external portions of the rods and cones set up nerve impulses.

158. Photochemical Theory of Vision.—Advocates of this theory claim that the ether waves produce chemical changes in the rods or cones. Hence the nerve impulses are due to chemical changes.

This theory is based upon a pigment contained in the retina, and known as the visual purple.

159. Visual Purple Rhodopsin.—The visual purple is a red pigment which is rapidly bleached by light. It is found only in the external segments of the rods. It is therefore absent in the fovea. If an eye is dissected in a ruby light and the retina detached, it is at first of a deep red color. If exposed to sunlight it soon becomes colorless. If this retina is laid back upon the pigmented epithelial cells, it soon regains its reddish color. An actual photograph has been taken upon a dissected retina.

The visual purple does not seem essential to vision as it is absent in the eyes of some day and night owls. It is claimed that it adds to the sensitiveness of the rods.

160. Rods and Cones, Function of.—It has been fairly well established that the function of the rods is to determine shades of light. Perception of colors is the function of the cones. We may infer then that color blindness is due to a lack of sensitiveness of the cones. *Night blindness* may be due to a defect of the rods. In case of defective vision we may look for a lesion of the optic tract fibers. Or a lesion in the cortical area.

161. Retinal Image, Size of.—If we hold before the eye two very fine objects and then bring them together until just before they appear as one, we will find that they subtend an angle of approx. 1, minute. On the retina the image is about .004 mm. diameter, which is the minimum.

162. Young-Helmholtz Theory of Color Perception.—In 1807 Thomas Young advanced the theory afterward modified by Helmholtz that we have three basic color sensations, *i.e.*, red, green and violet. Each color has a corresponding photochemical substance. These colors have their individual nerve fibers and visual centers. The sensation of white results when the three sets of fibers are equally stimulated.

This theory is not without fault. We might point out that the periphery of the retina represents a color blind area although we can distinguish white and grey. It also fails to account for partial and total color blindness.

163. Color Blindness. Daltonism.—Strange to say it did not receive much attention until Dalton, an English chemist, conducted some investigations at the close of the 18th century. Holmgren, a Swedish physiologist, also contributed much to our present knowledge.

The ability to distinguish different colors may be partial or complete.

The cause may be of a congenital origin, or disease. Particularly atrophy of the optic nerve. Continuous work on fast colors may bring about this condition.

If no color can be distinguished, the condition is termed *achromatic vision.* If but two or three primary colors *dichromatic vision.* Red blindness is the most commonly met with. Color blindness is found in from 2 to 4 per cent. of males, and but .01 to 1 per cent. in females. This abnormal condition exists to a greater extent among the illiterate.

164. The Retinal Field of Colors.—The peripheral portion of the retina is color blind and the perception is limited to shades of light. As the fovea is approached the color blue is first perceived; green last. Certain parts of the retina are therefore insensible to certain colors. Remarkable to say the perimeter does not record the same color fields in different persons of normal vision.

REFRACTION CHAPTER IV, ACCOMMODATION CHAPTER V, AND CONVERGENCE CHAPTER VI

The chapters Refraction, Accommodation and Convergence are to enable the refractionist to fit the eyes with glasses that will afford his patient the best and most comfortable vision. To accomplish this the following examinations must be made.

Refraction Chapter IV.—Examine the refraction of the eyes, to ascertain if parallel rays focus upon both retinas, with the accommodation at rest.

Accommodation Chapter V.—Ascertain if the accommodation is adequate, to keep the image of objects at varying distances, focused upon both retinas.

Convergence Chapter VI.—Ascertain if the muscles controlling the movements of the eyeballs are so balanced, that the image of objects, situated at varying distances, are kept upon the macula of each eye.

No examination can be considered complete, until after a thorough ophthalmoscopic and medical examination has been made. Ophthalmology does not come within the domain of this work.

CHAPTER IV

REFRACTION OF THE EYE

MERIDIANS OF THE EYE. TERMS DEFINED. SCHEMA. EMMETROPIA. AMETROPIA. AXIAL AND CURVATURE. SIMPLE HYPERMETROPIA. SIMPLE MYOPIA. CURVATURE AMETROPIA. IRREGULAR AND REGULAR ASTIGMATISM. PRESBYOPIA. ANISOMETROPIA. APHAKIA. WHEN SHOULD GLASSES BE WORN?

This chapter treats on the optical state of the eye, viz. causes of errors of refraction and how corrected.

165. Meridians of the Eye.—The meridians or axes of an eye can be read from the degree signs on the trial frame, as they coincide. The degrees are stamped upon the cell, which forms a semi-circle.

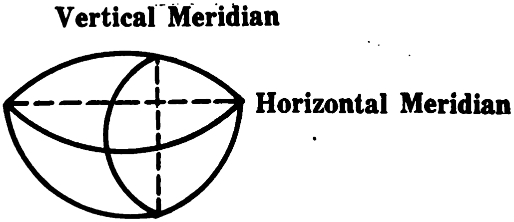


FIG. 108.

The trial frame should be so adjusted that the pupil will be at the point of intersection of the imaginary line from O, to O, and the perpendicular line from 90°. Figs. 108, 109, 110. The optical center of the trial lens, should occupy this position.

The numbering for both eyes starts on the left hand side of the trial frame at O, the horizontal meridian. The numbering is continued until a half circle is described (which includes 180°). If the cell and degree markings were continued, forming a complete circle, the degrees on the lower half would be opposite to the corresponding degrees on the upper half. Hence, it is unnecessary to have the cell describe a complete circle. Some trial frames have the upper instead of the lower part of the cell marked in degrees.

In astigmatism, the degree of one of the principal meridians is indicated by the degree at which the cylindrical axis stands, *e.g.*, in Fig. 109 the axis points to 45°.

FIG. 109.

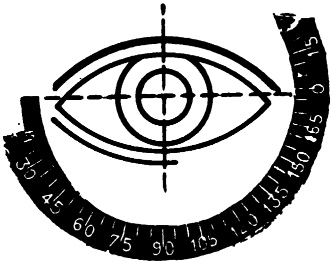


FIG. 110.

Terms "Twenty Feet," "Parallel Rays," "Distance," "Infinity"

166. Parallel rays, Distance, Infinity or its sign ∞ are terms used interchangeably by the refractionist. Because rays proceeding from an object situated at or any distance beyond twenty feet, are practically parallel, as if they came from infinity.

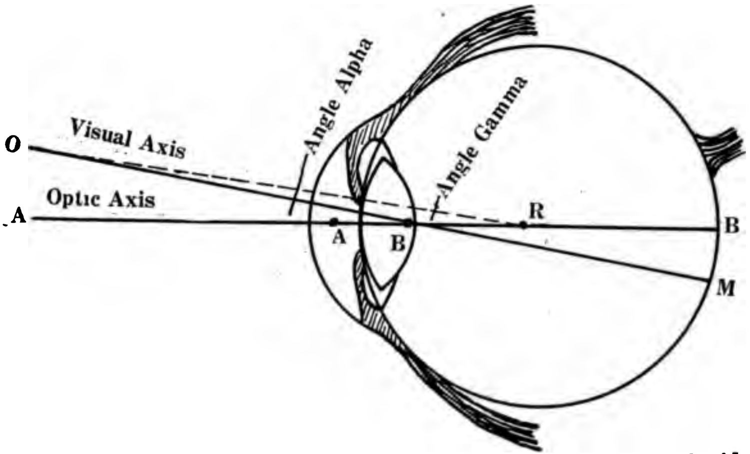
167. Schema Standard Eye.—The following schema is used where calculations are necessary. Fig. 111. The eye has three refracting surfaces, viz.: Anterior surface of cornea, anterior surface of crystalline lens, and anterior surface of the vitreous. The aqueous, lens and vitreous are the refracting media.

The dioptric media of the eye is of approximately 23 mm. focus. The six cardinal points are designated as follows: Two principal points, two nodal points and two principal foci.

The two principal points *A*, are approximately 2 mm. back of the cornea and almost coincide. The two nodal points *B*, are approximately 7 mm. back of the cornea and nearly coincide with the optical center of the refractive media.

The first principal focus is approximately 13.7 mm. At this point parallel rays from the retina would focus.

The second principal focus is located 22.8 mm. back of the cornea. At this point parallel rays are focused. The center of rotation *R*, is 9.8 mm. from the retina.



Some authors assume the angle O. R. A. to be the Angle Alpha

Fig. 111.

Emmetropia

168. Emmetropia, Ab.E.—Emmetropia: Gr. εν, *within;* μετρον, *measure; and* ωψ, *eye).—In an emmetropic eye perfect images of objects are formed upon the retina; if the objects are situated at twenty feet or farther, the accommodation being at rest.* Fig. 112.

This is because the length of the eyeball and strength of the refractive media are such that parallel rays passing through all meridians of the media are brought to a common focus upon the retina.

In hypermetropia, the image is formed back of the retina, and in myopia, in front of the retina. Therefore the image which is formed upon the retina is indistinct, so also is the vision.

Strictly speaking, emmetropia is a rare condition.

Emmetropia does not necessarily mean perfect vision. Because there may be opacities of the refractive media, which interfere with the passage of light, resulting in an imperfectly defined image. The nerves of the retina may also be impaired.

As in emmetropia, an eye should not accommodate for distant objects. Therefore to make the ametropic eye artificially emmetropic with lenses, is what the refractionist should strive for. This is accomplished by correcting the patient's vision for distance with lenses. Lenses of different strength are unnecessary for nearby vision, unless the accommodation is lessened through age or illness.

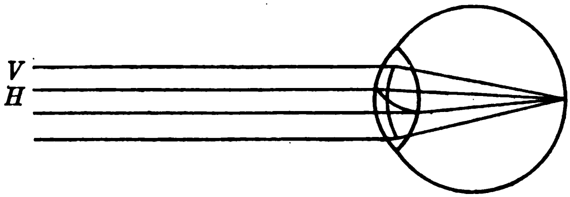


FIG. 112.

169. Objective Evidence.—Using the plane mirror at 40 in. and with a +1.00 D. lens in the trial frame (the accommodation suspended) the movement is neutralized. Retinoscopy, Sec. 328.

In direct ophthalmoscopy, no lens is required to render the details of the fundus clear.

In indirect ophthalmoscopy, the aerial image remains unchanged in size, as the convex lens is receded.

The three above methods prove the emergent rays to be parallel. The punctum remotum is at infinity, Fig. 113.

170. Subjective Evidence.—If the refractive media is clear, and there are no defects in the nerves the distant vision is good. Also nearby vision, unless the accommodation is lessened. Healthy young emmetropes frequently have a visual acuity above normal.

171. **Size of Images.**—*Punctum Remotum, Punctum Proximum, and Convergence.*—Fig. 85 proves that the retinal images are larger than in hypermetropia, and smaller than in myopia.

The punctum remotum (far point) is at infinity. The punctum proximum varies with age. See "Presbyopia," Sec. 216.

For "Convergence in Emmetropia" see Chapter IV.

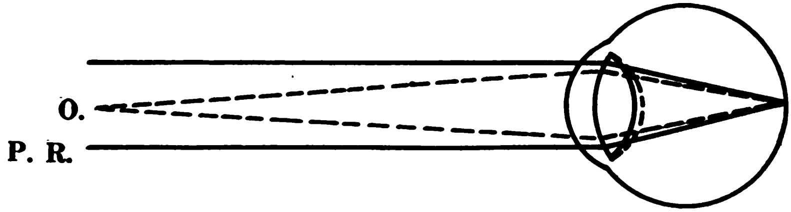


FIG. 113.

Ametropia

172. **Ametropia** (Ametropia: Gr. *α, privative; μετρον, measure; οψις, vision*).—*Ametropia is a condition of the eye (with the accommodation passive), in which parallel rays from an object, after passing through all meridians of the refractive media do not focus upon the retina.*

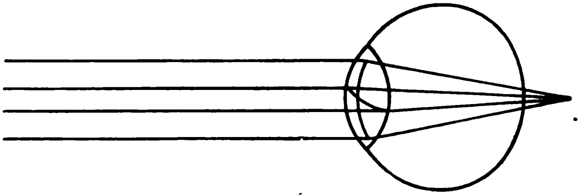


FIG. 114.

Ametropia is divided into Axial and Curvature Ametropia.

173. **Axial Ametropia. Simple Hypermetropia and Simple Myopia.**—*Axial ametropia is a condition in which parallel rays after*

passing through all meridians of the refractive media, focus at a point upon the optic axis, but not at the retina as in emmetropia.

In simple hypermetropia, the rays would focus at an imaginary point back of the retina, Fig. 114, and in simple myopia they focus in front of the retina, Fig. 115.

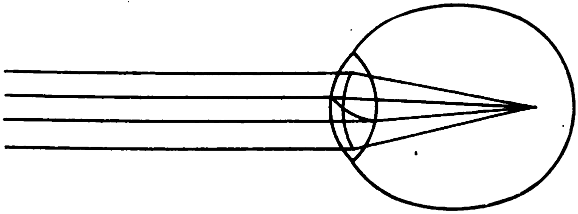


Fig. 115.

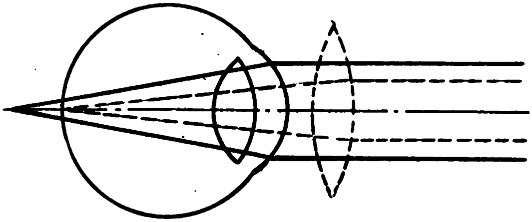


Fig. 116.

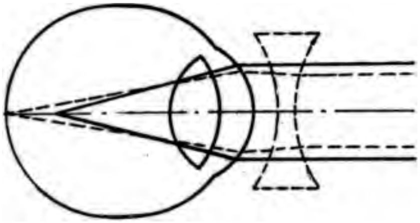


Fig. 117.

Hypermetropia is corrected by a plus lens, Fig. 116, and myopia is corrected by a minus lens, Fig. 117.

Bear in mind that a clear image is formed where the secondary axial ray crosses the refracted ray. From Fig. 85, it is plain that the point of intersection is back of the retina in hypermetropia, and in front of the retina in myopia. The image not being sharply defined the patient sees indistinctly. The greater the distance of this point from the retina the higher is said to be the ametropia.

This principle can be demonstrated with an ordinary camera: First, adjust the ground glass screen until a sharply defined image is obtained. This condition is identical with that of emmetropia. Next move the screen forward and you will note that the image decreases in definition. We now have a condition of hypermetropia. To demonstrate how well a myope sees move the screen back of the point of emmetropia. This indistinct image is referred to as circles of diffusion.

We may now demonstrate how ametropia is corrected by lenses. Set the camera at hypermetropia. Then place before the objective plus lenses until one is found that renders the image clearest. This lens is a measure of the hypermetropia. If the camera is set to represent myopia, minus lenses are to be used. The lens that renders the details of the image distinct is a measure of the myopia.

The schematic eye, Fig. 185, is constructed according to the above described principles.

174. Curvature Ametropia Astigmatism.—*Curvature ametropia, is a condition in which there is a difference in curvature of the different parts of the refractive media. Therefore the rays are not brought to a common focus as in emmetropia or axial ametropia.*

Curvature Ametropia is divided into Irregular and Regular Astigmatism.

175. Simple Hypermetropia or Axial Ametropia (Far Sight) Ab.H. (Hypermetropia: Gr. ὑπερ, *beyond;* μετρον, *measure;* and οψις, *vision*). *Parallel rays focus back of the retina, with the accommodation at rest,* Fig. 114.

This condition is almost invariably due to a short eyeball. The refractive power of the media may be insufficient.

176. Causes of Hypermetropia.—A hypermetropic eye is a short, undeveloped or congenitally defective eye. Therefore infants and children are naturally hypermetropic. If, in time, they do not become emmetropic, it is due to arrested development. Owing to the flexibility of the crystalline lens in the young, the focus can easily be brought to the retina; except in extremely short eyes, or in an eye where the ciliary muscle is undeveloped.

While by growth the eye tends to increase in length the crystalline lens and cornea gradually decrease in refractive power. From fifty to seventy-five years of age, the refractive media, due to flattening of the lens decreases in refractive power about 2.50 D. The index of refraction of the media sometimes diminishes as a result of disease, also tending toward hypermetropia.

Thus the myopic eye of a low degree may become emmetropic. And an emmetropic eye may become hypermetropic; designated "Acquired Hypermetropia."

Absence of the crystalline lens (aphakia) produces hypermetropia.

177. Accommodation in Hypermetropia.—An emmetrope, to see clearly, when looking at an object situated at infinity (distance) need not accommodate. But if the object was at thirteen inches, he must accommodate 3.00 D.

A hypermetrope of 2.00 D. for distance, must accommodate 2.00 D. in order to overcome his refractive error. To see clearly at thirteen inches, he must accommodate another 3.00 D. making a total of 5.00 D.

From the foregoing, it can be seen that presbyopic symptoms are manifested at an earlier age in hypermetropia than in emmetropia. The punctum proximum is also farther removed.

The ciliary muscle in hypermetropia is usually larger, than in either emmetropia or myopia. This may be congenital or brought about through the demand for extra work in accommodation.

178. Punctum Remotum (Far Point).—Fig. 118 is spoken of being negative, because rays from the retina emerge divergent, and none other than convergent rays would focus upon the retina (acc. at rest). The P. R. is located where these divergent rays would meet, if projected backward, P. R. About 80 per cent. of all eyes are hypermetropic.

The retinal image is smaller than in emmetropia or myopia.

For "Accommodation in Hypermetropia" see Chapter V.

For "Convergence in Hypermetropia" see Chapter VI.

179. Divisions of Hypermetropia.—On account of the different conditions of the ciliary muscle. Hypermetropia has been divided as follows: *Latent H.*, *Manifest H.* and *Total H.*

Manifest H.—Is the amount of hypermetropia which is corrected by a plus lens; the accommodation not being suspended.

It is subdivided into *Faculative*, *Relative* and *Absolute*. *Latent Hypermetropia* Abb. L. H. The amount that remains after the manifest has been corrected.

The latent is caused by a cramp or contraction of the ciliary muscle. It can be measured only after the ciliary muscle has been relaxed. A reliable cyclopegic is positive. Where latent H. is present, the patient may manifest evidence of emmetropia or myopia. Usually before the age of forty, the latent has become manifest.

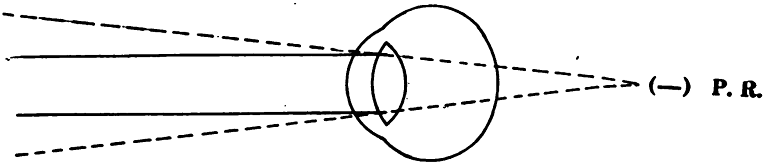


FIG. 118.

Total Hypermetropia.—The sum of the Latent and Manifest, *e.g.*, Manifest +2.00 D. Latent +1.00 D. = 3.00 D. Total H.

Faculative Hypermetropia.—*Voluntary H.* Distant and nearby vision good without lenses. Indicative of adequate accommodation.

Relative Hypermetropia.—Is a condition in which the patient overcomes his refractive error by converging to a point, which is at a less distance than the object looked at.

There is an appearance of convergent squint (Esophoria). Plus lenses should be prescribed.

Absolute Hypermetropia.—The patient cannot overcome his refractive error by accommodating: *e.g.*, if the H. was 5.00 D. and the accommodation but 3.00 D., the patient could not see distinctly at any distance. This occurs in presbyopia. Bi-focals, should be prescribed for constant wear. *Acquired H.* due to natural flattening of the lens, and diminished index of refraction. Also removal of the lens in aphakia.

Symptomatology

180. Objective Symptoms.—In many cases the face is narrow, pupillary distance short.

In Retinoscopy, plane mirror +1.00 D. lens; working distance 40 in., "movement with mirror."

In Direct Ophthalmoscopy, plus lens renders details of fundus clearest.

In Indirect Ophthalmoscopy, size of aerial image decreases as lens is receded.

The three above methods prove the emergent rays to be parallel, which is indicative of hypermetropia.

The sclera may appear irritated. Frequently tears. Many young Hypermetropes hold the reading closer than thirteen inches, to secure a larger (and if the illumination is faulty a brighter) retinal image.

181. Subjective Symptoms.—If the accommodation is adequate distant and nearby vision is good.

Upon being questioned, the patient states that after using the eyes for nearby work, in some cases distance also, different symptoms of asthenopia develop. Plus lenses improve vision.

182. Glasses for Hypermetropia.—The treatment consists of prescribing the strongest plus lenses that will afford the best and most comfortable vision, for distance and nearby. This may necessitate the use of atropine or bi-focals.

On account of excessive accommodation there may be a disturbance between the two functions, accommodation and convergence, unless the correction is worn constantly.

As there is apt to be asthenopia, while *latent H.* remains, it is advisable to correct it soon as possible. Hence the reason for prescribing the strongest plus lens. First test with plus lenses when the eyes are not under the influence of a cycloplegic. This obtains an approximate measure of the *Manifest H.* Make a record. Then relax the accommodation (a reliable cycloplegic is positive). Make another test to ascertain the latent.

Just what per cent. of the latent can be corrected at once with benefit, cannot be stated as a rule, as in different individuals, it varies. In some cases no more than the manifest can be corrected, even after the patient has been atropinized for several days. In other cases, but a portion of the latent can be corrected; in others, the *total* may be corrected.

For treatment See "Cycloplegic," Chapter VII.

183. Simple Myopia or Axial Ametropia (Near Sight) Ab. M. (Hypometropia or Brachymetropia: Gr. *υπο*, *under*, or *βραχυς*, *short*, *μετρον*, *measure*, *ωψ*, *eye*).—*Parallel rays focus in front of the retina, with the accommodation at rest, Fig.* 115.

This condition is also termed hypometropia, or brachymetropia.

The myopia is almost invariably due to a long eyeball, but may be due to the curvature of the refractive media, being too sharp.

184. Causes of Myopia.—It is usually an acquired condition. The elongation is due to stretching of a weakened sclerotic coat; rarely of the cornea. It occurs almost always at the posterior end of the eye ball. It is brought about as follows: Increased intraocular pressure from venous congestion within the eye, which congestion is due to interference with the return circulation through the neck, from such causes as inclining the head too much forward; the incorrect position often assumed in reading; tight collars, etc.

Convergence is also an important factor. It causes the internal recti muscles to pull upon the globe, pressing it against the surrounding tissues. This also increases the intra-ocular tension.

As the eyeball increases in length, making the eye still more myopic, the amount of convergence must increase proportionately, in order that the image falls upon the macula of each eye. Also the longer the eye ball, the more difficult is its rotation in the orbit.

Wide interpupillary distance, also makes necessary added convergence for nearby work.

Excessive convergence greatly increases the intraocular pressure and is supposed to cause a pulling outward of the sclera, by the optic nerve membranes, which are attached to the sclera, at the entrance of the optic nerve.

The above mentioned conditions, produce an increased tension about the globe, thus interfering with the nutrition of the tunicas. Stretching of the sclera, naturally results when increased tension is associated with the following conditions:

Congenitally thin sclerotic coat, probably the predisposing cause in cases ascribed hereditary.

Sclerotic rendered weak through general innutrition, resulting from exhausting diseases, fevers, etc.

Sclerotic softened by an inflammatory process.

Workmen who use a strong magnifier over one eye, and neither accommodate or converge, show no increase in the length of the eye ball, from their nearby work.

Ulcers and other inflammatory conditions of the cornea, sometimes weaken it to such an extent, that the internal pressure causes it to bulge. *Anterior Staphyloma.*

185. **Conic Cornea.**—In a healthy condition of the cornea, where bulging occurs, it is at the posterior end of the globe, around the optic disc.

Posterior Staphyloma.—Both anterior and posterior staphyloma increase the length of the eyeball. Anterior staphyloma also increases the sharpness of the corneal curve of the staphylomatous area, causing the rays to focus forward still farther. The surrounding peripheral portion of the cornea is flatter. This accounts for the peripheral ring, observed in retinoscopy.

Increased index of refraction or swelling of the crystalline lens in a cataract formation.

186. Stationary Myopia.—Myopia increases most rapidly between the ages of 7 and 14. It usually stops before the adult age, then termed stationary myopia. But under conditions previously described it may increase, resulting in serious impairment of vision.

Progressive myopia is usually ascribed as the result of subacute inflammation of the tunicas.

187. Progressive (Pernicious) Myopia.—Usually ascribed to softened tunicas, the result of a low grade of inflammation.

Myopia that does not become stationary at adult age is indicative of a pernicious type, and may continue to the serious impairment of vision.

Progressive pernicious myopia is usually due to a specific cause.

But without inflammation, where the sclera is probably thin, this condition may be excited by the stooping posture, continuous work by faulty illumination, at an unusually short distance, which requires excessive convergence.

Where the refractive medium is not clear, or the sensitiveness of the retina is diminished, the tendency to hold the work close is greater than ever. This lessens the possibility of arresting its progress. Because the vision cannot be improved much with lenses, and to see most distinctly the work must be held close.

Symptomatology

188. Objective Symptoms.—Many myopes have a broad face and wide interpupillary distance, but the features may be the reverse.

Many myopes close the lids to a narrow slit. This is done to shut off the peripheral portion of the media, or by lid pressure to overcome astigmatism.

In retinoscopy, plane mirror, working distance 40 in. "+ 1.00 D. lens, movement against mirror."

In direct ophthalmoscopy minus lens renders details of fundus clearest.

In indirect ophthalmoscopy, size of aerial image increases as the lens is receded.

The three above methods prove the emergent rays convergent, which is indicative of myopia. The eyes may appear to converge. Often limitation of the peripheral field of vision. Sometimes meta-morphopsia when looking at distant objects. Straight edges appear curved. This is claimed to be due to displaced retinal cones.

189. Ophthalmoscopic Appearance of Fundus.—The fundus may appear normal. Plate III. Or at the outer side of the disc may be observed a crescentic shaped, whitish area, *Myopic Crescent*, Plate I. Or the atrophied area may completely surround the disc, *Annular Staphyloma*, Plate II. Vessels over atropied area usually smaller and straighter. Sometimes they have disappeared.

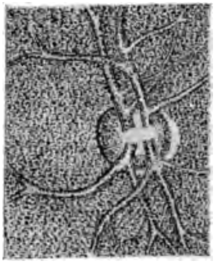


PLATE I


PLATE II


PLATE III

Over the staphylomatous area, the choroidal pigment may disappear in places, leaving the white sclerotic coat exposed to view. Or the whitish patches may be due to choroiditis with lengthening of the eyeball.

Staphyloma more frequently occurs on the temporal side of the disc, tilting it outward. Because of being seen obliquely, it appears oblong, one edge being fainter, and requires a lens of different strength to see the bottom of the "cupped" area.

In the pernicious or inflammatory type. Edges of crescent appear blurred from congestion. Atrophy increases. Vitreous turbid. Myopia increases. Failing vision. The eyes are irritable.

At one edge of the disc in some eyes may be seen pigment. This does not indicate a pathological condition. Plate I.

Often limitation of field of vision.

190. Subjective Symptoms.—As a rule, near vision is better than distant.

Reading usually held close.

Plus lenses blur; minus lenses improve vision.

Asthenopic symptoms commonly met with.

If there is squint, it is usually divergent (exophoria).

Often there is an intolerance of light.

Especially in ill health patient complains of small floating specks, *muscæ volitantes*.

Perimeter in many cases indicates limitation of field of vision.

Punctum Remotum (Far Point); Punctum Proximum (Near Point); Accommodation and Convergence

191. Punctum Remotum (Far Point).—Fig. 119. The retina being situated beyond the principal focus of the refractive medium rays

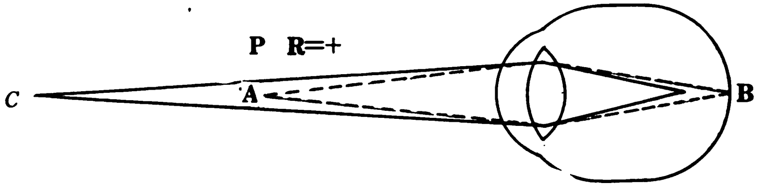


FIG. 119.

from it emerge convergent, and meet at a point *A*, the P. R. or *vice versa*. *A*, and *B*, are therefore conjugate. *A*, marks the greatest distance of the myope's most distinct vision. As the distance beyond the P. R. increases, objects grow less distinct. Because rays from them would strike the cornea less divergent and focus in front of the retina. See heavy lines from *C*. The myope must accommodate for objects situated at less than the P. R.

The higher the myopia, the closer is the P. R. The less the myopia, the farther away is the P. R. The distance of the P. R. corresponds to the focal length of the concave lens that corrects the myopia, *e.g.*, myopia 5.00 D. P. R. is at a distance of eight inches. This is the focal length of a 5.00 D. lens.

Fig. 120. Parallel rays *A*, focus at *B*, before they impinge upon the retina. Diverging rays from a nearby object at the exact distance will focus upon the retina. See Sec. 244.

If the myopia was 3.00 D. rays from an object at 13 inches (distance of P. R.) would focus upon the retina. If less than 3.00 D. the rays would focus back of the retina, and to obtain best vision the myope would have to accommodate.

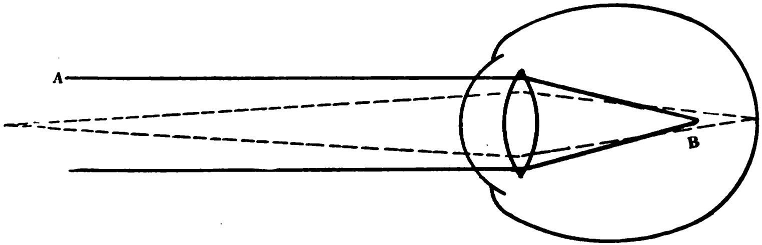


FIG. 120.

192. Punctum proximum (near point) is the shortest distance at which the myope can distinctly see an object, *e.g.*, No. 1 Type.

The range of distinct vision lies between the *far* and *near* points. It is less than in emmetropia, or hypermetropia, because the myope is helpless to accommodate for objects beyond the *far point*.

The amplitude of accommodation is usually below normal, because the myope does not accommodate for distant objects, and need not put into force as much as the emmetrope, or hypermetrope, for nearby work. The result is an undeveloped ciliary muscle through lack of use. In high degrees it may even lack the circular fibers; or it may have atrophied or been congenitally small. The strength of the ciliary muscle has much to do with the glasses that will be comfortable.

The cornea may be less convex than in emmetropia or hypermetropia.

The angle Alpha is smaller than in emmetropia or hypermetropia. The angle Gamma is sometimes negative.

Few myopic eyes are free from astigmatism.

Myopia is claimed to be on the increase.

193. Treatment of Myopia.—As the object of treatment is to improve the vision and prevent an increase of the myopia, the practitioner should take pains that the following unfavorable conditions are avoided. The stooping posture in reading. Faulty illumination, either too bright or too dim. Holding the work closer than thirteen inches. Persistently using the eyes when they become tired; calendered paper; fine type, etc.

The light should not shine in the patient's eyes but come over the left shoulder, so that the hand in writing will not cast a shadow upon the work. If the patient is left handed the light should come over the right shoulder.

194. Treatment of False Myopia.—*Myopia due to excessive contraction of the ciliary muscle.* Usually found in young persons, or persons having a hypersensitive nervous system, who may be emmetropic or hypermetropic. Thoroughly relax the ciliary muscle. Prescribe for constant wear such lens (weakest) that afford comfortable vision. Frequent changes of lenses is necessary in some cases. Examine the ocular muscles for insufficiencies.

195. Treatment of Progressive Myopia.—Progressive pernicious myopia, resulting from subacute inflammation of the tunica. Its specific cause, if possible, should be ascertained.

The treatment usually prescribed consists of complete rest to the eyes for several months. Outdoor exercise. Smoked plano lenses. Artificial leech.

If the vision shows signs of improvement, the eyes may be exercised a little daily, the correcting lenses being worn. Medical treatment as indicated. The results are usually discouraging.

196. Glasses for Myopia.—In myopia, to obtain perfect and comfortable binocular vision, both for distance and nearby with the same lenses, is often difficult, and sometimes impossible. This may be due to weakened accommodation or convergence or both.

The myope who can use his distant correction for nearby work with comfort probably has well developed ciliaries and recti muscles. If there is no evidence of past or present progressive myopia, these lenses should be worn constantly. This will establish the normal relationship between accommodation and convergence, *i. e.*, with each diopter of accommodation, one degree of convergence is made.

The myope who cannot comfortably wear the distant correction for nearby work, probably has a weak ciliary or rectus muscle. Both the amplitude of accommodation and convergence should be examined to ascertain where the fault lies. See tests for "Amplitude of Accommodation," Sec. 238 and "Convergence," Sec. 254, 255.

If the accommodation is inadequate, glasses weaker than the distant correction by about one or two diopters should be worn for nearby work. This is to avoid over taxation of the weak ciliary, and to exercise it as in paresis. In some cases the ciliary increases in tone, and the strength of the lenses for nearby work may be increased from time to time, until finally the distant correction will be suitable for all distances.

If the convergence is at fault see "Decentration of Lenses," Sec. 286.

Where, by test, the accommodation and convergence are found adequate, and the distant correction causes discomfort when used at the reading distance it is probably the result of long dissociation of accommodation and convergence. Or the discomfort may come from a hypersensitive retina. With the distant correction worn, the normal relationship between accommodation and convergence is established. To become accustomed to this, it may require time.

In cases where no accommodation is allowable the reading glasses must be 3.00 D. weaker than the full distant corrections, if the working distance is thirteen inches.

In high degrees of myopia, the full correction improves the details of the retinal image, but decreases its size. For this reason, the patient may complain that objects appear smaller in size. This objection the refractionist should over-rule, by explaining that this is the result of the proper correction, and that in time it will cease to be noticeable.

If the pupillary distance is extra wide, or the eyeball long the lenses can sometimes be decentered to make up for the extra convergence.

In myopia, the focus is in the vitreous, resulting in a diffused, less bright retinal image. Thus kept in the fog, some retinas become hypersensitive to a sharp focus or bright light. Some of these patients never can wear their full distant correction with comfort. By an occasional change of lenses from weaker to stronger, others can eventually wear their full correction.

In all cases of myopia, the patient should have a complete record of his test. So that in subsequent tests the progress of the myopia, if any, may be noted, should the patient for any reason go to different refractionists.

Curvature Ametropia, Astigmatism

197. Astigmatism Abb. A. (Astigmatism: Gr. *α*, *privative*, and *στιγμα*, *a point*).—*Astigmatism is a condition in which there is a difference in curvature of the different meridians of the refractive media.*

Rays passing through all meridians do not converge to a common focus on the optic axis, as in emmetropia and axial ametropia.

Astigmatism is divided into *Irregular* and *Regular Astigmatism.*

198. Causes of Astigmatism.—Astigmatism, as a rule, is a congenital condition. It has been known to run through several generations of the same family, which indicates hereditary tendency. Usually the cornea alone is involved.

It may be *acquired.* In rare cases the crystalline lens is displaced at birth. Also after birth, from injury. The reclining posture in reading is ascribed as a cause of tilting of the lens.

Index of refraction changed by cataract formation. Irregular corneal curvature results from extraction for cataract, iridotomy, tenotomy, or accidental wounds, scars and conic cornea, resulting from ulcers and keratitis.

199. Transient, Functional Physiologic Astigmatism is caused by excessive contraction of the lids, which has a tendency to change the shape of the corneal surface. Or the ciliary muscle may not contract equally the different sectors of the lens.

Or the lids sometimes closing to a narrow fissure attract the secretions, forming a minus cylinder, with its axis horizontal.

From the foregoing, it can be seen how *real astigmatism* may be concealed by this muscular contraction. Also through contraction of the iris to a pin hole opening, when the examination room is too brightly illuminated.

The principal divisions of astigmatism are *Irregular* and *Regular*, which are classified according to the curvature of the refractive media.

200. Irregular Astigmatism.—This condition is due to displacement or disease of the crystalline lens, and often irregularities of the cornea, caused by disease and wounds. Thus it will be seen that a meridian is not of equal refractive power in all its parts. Therefore, rays passing through all meridians usually cannot be brought to a common focus.

The success of a correcting lens will depend upon the nature and location of these abnormal conditions. A ground glass stenoptic lens will sometimes give much improvement in the latter condition, if the opening, the size of which has been ascertained in the test, is placed over the part nearest normal.

Astigmatism Caused by Cataract Operation.—In some cases after the lens has been removed the cornea will have 14 D., of astigmatism. In a couple of months it will recede to 1.00 D. Irregular corneal astigmatism may persist.

201. Regular Astigmatism.—In regular astigmatism the cornea instead of being spherical as in emmetropia and axial ametropia, is practically spoon shaped (the segment of an ellipsoid). Therefore, the curve of one meridian vertical, Fig. 108, is sharper than the one horizontal, which is at right angles to it. These are the meridians of *least* and *greatest curvature*, and are spoken of as the *two principal meridians*. They are usually at 90 and 180 degrees. The flatness of the cornea in the 180th meridian may be due to constant lid pressure.

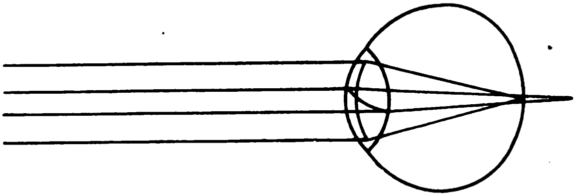


FIG. 121.

The Stenopaic Slit Fig. 164, is of much aid in locating these meridians.

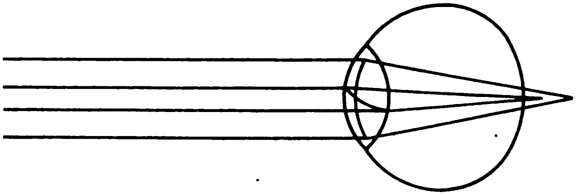


FIG. 122.

Regular astigmatism is subdivided according to the location of the focus of the two principal meridians: Simple Hypermetropic, compound Hypermetropic, Simple Myopic, Compound Myopic and Mixed Astigmatism.

See "Optics," Chapter 11, Action of cylinders and sphero cylinders.

202. Simple Hypermetropic Astigmatism.—Rays from one principal meridian, focus upon the retina, while rays from the meridian at right angles to it, focus back of the retina, Fig. 123.

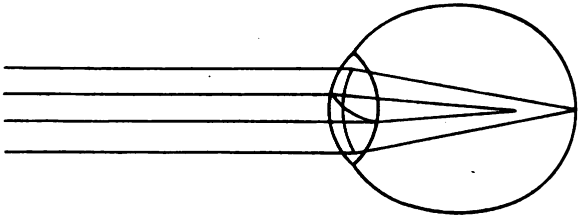


FIG. 123.

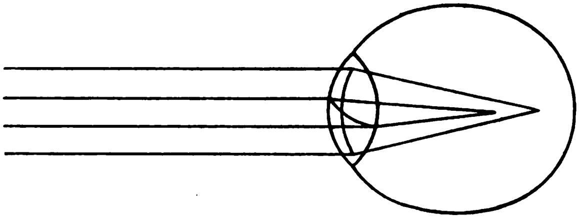


FIG. 124.

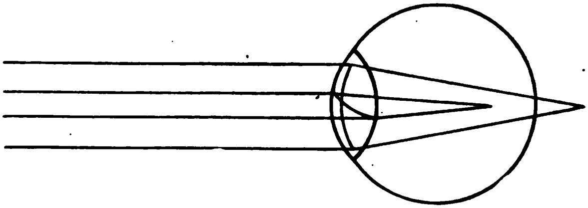


FIG. 125.

203. Compound Hypermetropic Astigmatism.—Rays from both principal meridians focus back of the retina, but at varying distances, Fig. 125.

204. Simple Myopic Astigmatism.—Rays from one principal meridian, focus upon the retina, while rays from the meridians at right angles to it, focus in front of the retina, Fig. 127.

205. **Compound Myopic Astigmatis.**—Rays from both principal meridians focus in front of the retina, but at varying distances, Fig. 124.

206. **Mixed Astigmatism.**—Rays from one principal meridian focus in front of the retina (myopic) while rays from the meridian at right angles, focus back of the retina (hypermetropic), Fig. 124.

207. **Symmetric and Asymmetric Astigmatism.**—If by adding the degrees of the cylindrical axes of each eye, the result is 180° it is a case of *symmetric* astigmatism. If the result is not 180° the condition is spoken of as *asymmetric* astigmatism, *i.e.* the two principal meridians are not at right angles.

208. **Astigmatism with the rule** is indicated when the correcting cyl. is minus and its axis nearer the 180th meridian than the 90th. Or if the cyl. is plus with its axis nearer the 90th than the 180th.

209. **Astigmatism against the rule** is indicated when the correcting cyl. is plus and its axis is nearer the 180th meridian than the 90th. Or if the cyl. is minus with its axis nearer the 90th than the 180th.

210. **Normal Astigmatism.**—In the majority of eyes the radius of curvature of the vertical meridian of the cornea, is slightly shorter than the radius of the horizontal meridian. The crystalline lens if of shorter radius in the horizontal meridian. The effect is that of cross cylinders, but that of the crystalline lens does not quite neutralize the normal astigamtism of the cornea. When the astigmatism exceeds 100 D. it is considered pathological by some writers.

This can be demonstrated by the following experiment: *Make a right angle cross on a white card. Close one eye and hold the card before the other. On moving it away, the horizontal line can be seen more distinctly at a shorter distance than the vertical line.*

Conic Cornea (*Anterior Staphyloma*). Bulging of a limited area of the cornea by stretching, due to softening from keritis or ulceration. If unnoticeable to the naked eye, it can be diagnosed by the retinoscope. See Chapter IX.

In correcting this error, if centrally located, the correction should be for the staphylomatus area. Sometimes the entire cornea can be corrected by lenticular lenses.

211. **Irregular normal lenticular astigmatism,** *is described as a normal condition of the crystalline lens.* It is said to be caused by a difference of index of refraction of the different sectors of the lens. As proof, a small round light at a distance appears stellate. On removal of the lens (aphakia) the light appears round.

Objective Symptoms

212. **In asymmetric astigmatism,** there is usually a lack of symmetry about the face or skull. In *Symmetric Astigmatism* the face and skull are usually symmetrical.

Wrinkles about the brows and eyes result from the muscular action to shape, the cornea.

Some astigmatic persons on looking turn the head sideways.

For diagnosis by "Retinoscopy," see Chapter IX.

For diagnosis by "Ophthalmoscopy," see Chapter X.

213. **Subjective Symptoms.**—As a rule, astigmatic patients see one set of lines on the clock faced chart plainer than the lines at right angles to them. See "Subjective Test," Chapter VIII.

Vision usually subnormal.

Few astigmatic persons are free from asthenopic symptoms.

214. **Peculiarities of Astigmatism.**—Seldom are hypermetropes and myopes free from astigmatism.

Usually the vertical meridian of the cornea is of shorter radius than the horizontal, but it may be the reverse.

Almost always there is a difference in the refraction of the two eyes.

215. **Treatment of Astigmatism.**—As in axial ametropia, the rule is to prescribe such lenses as will cause parallel rays to focus upon the retina.

In simple hypermetropic and simple myopic astigmatism a cylinder alone is sufficient. But in compound and mixed astigmatism, a sphero cylinder or cross-cylinder combination is necessary.

In asymmetric astigmatism the visual acuity is generally subnormal and arriving at the proper correction is difficult and sometimes impossible.

At least two tests should be made, separated by an interval of one half, to a day or more, according to the effect on the patient, as frequent changes of lenses in making a test sometimes cause vertigo, nausea, vomiting and also headache, lasting a day or so.

In subsequent tests, first place in the trial frame the combinations found best suited in the previous test. In this test slight alterations in the strength of the sphere, or cylinder, or its axis may be necessary to obtain the proper correction. Finally, before ordering the lenses ground, the correcting combination should be transposed in the trial frame and the patient allowed to read with them for half an hour or so.

Presbyopia

216. Presbyopia Abb. P. (Presbyopia: Gr. *πρεσβυς, an old man,* and *οψις, vision).—Presbyopia is generally agreed to be a condition in which the crystalline lens has become so hardened, due to advancing age, that an eye cannot accommodate to eight, nine or ten inches.* Authors do not agree upon an exact distance.

In infancy the lens is so elastic that its form can easily be changed (its refractive power increased several diopters).

With advancing age, its firmness constantly increases. At the age of forty-five, it has become so hardened, that its refractive power can be increased but 3.50 D. with the ciliary muscle in its highest state of contraction. With the ciliary in this state, it would soon become exhausted and relax. Usually at the age of forty an emmetrope must either wear a plus lens for reading, or hold the work farther away. When this occurs they are said to be presbyopic.

For comfortable reading at 13 inches 4.50 D. of accommodation is usually necessary.

217. Cause of Presbyopia.—Presbyopia is due to hardening of the crystalline lens, with advancing age. It is a natural hardening, which takes place in all eyes, at about the same rate, from infancy to seventy-five, when the accommodation becomes 0. Slight weakening of the ciliary muscle may also be a contributing factor. There are exceptions to this where individuals reach a greater age with vision very little affected. (Also Second Sight.)

Sclerosis of the lens in cataract formation and weakness of the ciliary from debilitating diseases, fevers, etc. are productive of lessened accommodation.

218. Objective Symptoms.—The patient if emmetropic, hypermetropic or myopic of a low degree, sees well at a distance; but in order to see clearly at a short distance he holds the reading farther away than formerly.

Prolonged reading, especially by artificial light is accompanied by fatigue to the eyes, and in many cases drowsiness.

219. Subjective Symptoms.—The test for accommodation Chapter V demonstrates the amplitude to be less than 3.50 D.

Patient if hypermetropic, in reading at 13 inches, can see better with a stronger plus lens than the distant correction. If myopic it is the reverse.

220. Glasses for Presbyopia.—The approximate nearby correction can be found by adding to the distant correction, 1.00 D. for every five years after forty-five up to the age of sixty.

If the distant vision is sub-normal bi-focals or two pairs, will be necessary, one for distance and one for nearby.

If the vision cannot be brought to normal, it may be due to the natural loss of transparency of the media or lack of sensitiveness of the nerve-fibers. Or the cause may be a pathological one.

221. Anisometropia.—Anisometropia (an-is-o-me-tro′-pe-ah) *Is a condition in which the refraction of one eye differs from that of the other.*

One eye may be emmetropic and the other ametropic, or both may be ametropic.

222. Causes.—Anisometropia is usually congenital. It may be acquired; the refraction of one or both eyes having become changed as a result of aphakia, staphyloma, tenotomy, hardening, etc.

Usually where the difference in the refraction of both eyes is not very great, they accommodate and converge equally, *binocular vision* being the result. One image is less distinct. Possibly some are able to disassociate the accommodation slightly, thus rendering both images distinct.

If the eyes are used *alternately*, usually one eye is emmetropic or hypermetropic, and the other myopic. The former being employed for distance. The latter for nearby vision. If one eye is used *exclusively* it may be found to be emmetropic or slightly hypermetropic, and the other highly myopic. The excluded eye may deviate.

223. Treatment.—If the two eyes are given their full correction, one image will be larger than the other, and the prismatic effect of the two lenses in converging will differ.

Ophthalmologists state that each case is a law unto itself.

Some will accept the full correction, others but a partial correction.

Alex. Duane, after working out a number of cases concludes that usually: The correction for each eye can be given successfully, even where the difference exceeds 2.00 D. The patient should be advised that it may require one or two weeks to become accustomed to the lenses.

If temporary discomfort is produced by the lenses it will disappear in from one hour to two weeks if the lenses are worn constantly.

When once accustomed to, they relieve symptoms which a partial correction would not.

Especially beneficial in beginning squint due to the anisometropia.

224. Aphakia.—Aphakia: *absence of the crystalline lens.*

225. Causes of Aphakia.—It is usually due to extraction for cataract. Dislocation, spontaneous or from injury, or absorption, due to injury, are also causes.

226. Symptomatology.—Lack of accommodation. Extra deep anterior chamber. Unusually black pupil. Oscillation of iris. Absence of lens, reflex.

When making the diagnosis by reflected light see Sec. 229. Acc. Chapter V, the images formed by the anterior and posterior surface of the crystalline lens are absent. Ophthalmoscope reveals traces of the opaque lens capsule.

227. Glasses for Aphakia.—The crystalline lens has a refractive power of about 19.00 D. Its absence therefore leaves the eye highly hypermetropic. In many cases this abnormal condition is overcome by contraction of the iris to a pin hole opening, obtaining the pinhole camera effect, so that reading can be done without lenses.

But usually an eye otherwise emmetropic requires about a 10.00 D. plus lens (commonly called "cataract lens") for distant vision. The reading glass is found as follows: Add to the distant correction a glass the focal length of which corresponds with the distance at which the patient is to work, *e.g.*, working distance 13 in. The focal length of a 3.00 D. lens is 13 in. Thus the reading glass is + 13.00 D.

An artificial accommodation can be obtained for near by work, by resting the lenses upon the nose at varying distances from the eyes.

Removal of the lens results in *normal astigmatism* becoming *apparent*. Irregular astigmatism may result from damage to the cornea.

228. When Should Glasses be Worn.—On theoretical grounds, when parallel rays do not focus upon the retina, the accommodation being passive. From the foregoing, it would appear that all ametropes should wear glasses constantly, which correct the distant vision. However the different conditions which are encountered in hypermetropia, myopia and astigmatism, deserve careful study. It should be borne in mind that in the great majority of cases the patient will derive more benefit if the lenses are worn constantly.

When a patient consults the refractionist concerning his eyes, it is usually because discomfort in some form accompanies their use. The relief from such symptoms in the great majority of cases is gratifying indeed. The following is a partial list of conditions which have been abated by properly fitted lenses: asthenopia, poor memory mental aberration, chronic conjunctivitis, styes, heterophoria, even heterotropia, gastric derangements, nausea, headache, vertigo, etc.

CHAPTER V

ACCOMMODATION

ACCOMMODATION THEORY OF. CHANGES DUE TO AGE. AMPLITUDE AT VARIOUS AGES. SPASM. PARALYSIS. PUNCTUM PROXIMUM. NEAR POINT. PUNCTUM REMOTUM. FAR POINT. RANGE. AMPLITUDE. ACCOMMODATION IN EMMETROPIA. HYPERMETROPIA. MYOPIA. PRESBYOPIA. ANISOMETROPIA AND APHAKIA

229. Theory of Accommodation.—Helmholtz, by utilizing the images of Pyrkinge demonstrated that during the act of accommodation the crystalline lens becomes more convex. The patient is taken into a darkened room and instructed to stare into space, Fig. 126. The surgeon takes a position a little to one side of the line of sight, and holds a small lighted candle a little to the other side. He will now observe three images. Fig. 127, *A*, is an erect, larger and comparatively fainter image reflected from the convex surface of the lens. *B*, is a smaller and much brighter image reflected from the convex surface of the cornea. *C*, is inverted, smaller and still

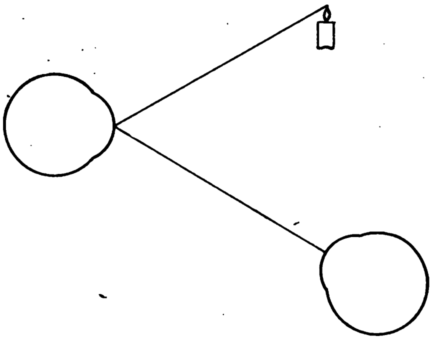


FIG. 126.

brighter. It is reflected from the posterior surface of the lens which acts as a concave mirror.

Next, instruct the patient to accommodate for a near by object, Fig. 128. It will be observed that the corneal image *B*, remains unchanged. The image *C*, changes but little which sustains the theory that the lens changes but slightly at the posterior side. The large image *A* reduced in size has moved closer to the image *B*. The diminution of *A*, proves the anterior surface to be of increased curvature. As further proof you are referred to the size of images as formed by a convex mirror, *i.e.*, the sharper the reflecting curve the smaller must be the image.

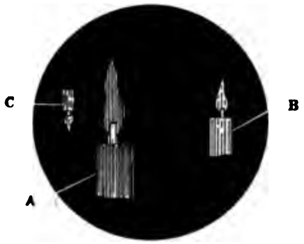


FIG. 127.

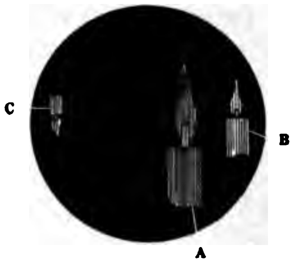


FIG. 128.

How the Curvature of the Lens is Brought About.—The lens hangs suspended by its ligament, under sufficient tension to flatten it. When the ciliary muscle contracts the circular area around the lens is decreased. The suspensory ligaments having lessened the tension the crystalline lens expands due to its inherent elasticity. The choroid is pulled forward. This allows the lens still more opportunity for expansion. Helmholtz Theory.

Accommodation consists of altering the focal length of the crystalline lens in order that the image of objects situated at varying distances will focus upon the retina. This is accomplished by contraction of the ciliary muscle. To demonstrate that an eye must accommodate for objects at various distances perform the following simple experiment. *Hold a finger a few inches before the eye and look* *at some object several feet away. The farthest object will appear distinct while the one nearby will be indistinct. Then look at the nearby object; it will be distinct, the distant one indistinct.*

This demonstrates that unless the refractive power of the crystalline lens is changed, rays of light from objects at varying distances will not focus upon the retina. If the accommodation is insufficient it is to be aided by lenses.

The dioptric power of the refractive media is usually stated at + 58.00 D. That of the crystalline lens about 15.00 D. ciliary muscle at rest. At 10 years of age its power can be increased about + 14.00 D. by contraction of the ciliary muscle. The lens loses its flexibility due to natural hardening until at the age of about seventy-five no change in its curvature is recognized, although the ciliary muscle probably makes practically the same effort as in former years. In the case of some old persons the lens becomes liquefied, which produces *second sight.*

The ciliary may be in a weakened state, as a result of debilitating fevers and diseases, such as typhoid, diphtheria, rheumatism, syphilis, etc. Or it may be congenitally so, or rendered weak from overtaxation. It is also found in a state of excessive contraction, also paralyzed. Such conditions may be referred to as *errors of accommodation.*

The following table gives the amount or amplitude usually present at the various ages.

THE AMOUNT OF ACCOMMODATION USUALLY PRESENT AT THE VARIOUS AGES

Age in years	Amplitude (Diopters)	Age in years	Amplitude (Diopters)
10	14.00	45	3.5
15	12.00	50	2.5
20	10.00	55	1.75
25	8.5	60	1.0
30	7.0	65	0.75
35	5.5	70	0.25
40	4.5	75	.0

Spasm of Accommodation

230. Spasm of Accommodation.—*State of excessive contraction of the ciliary muscle.*

Spasm is a very frequent occurrence in children and young adults.

In testing the refraction, unless the ciliary is relaxed (a cycloplegic is positive) the optical condition of an eye cannot always be ascertained.

In emmetropia, this condition would make the eye myopic, *false myopia*. And the hypermetropic eye less hypermetropic. The amount concealed by the ciliary muscle is termed the *latent hypermetropia*. That amount which manifests itself without relaxing the ciliary is designated the *manifest hypermetropia.*

The sum of the latent and manifest is termed the *total hypermetropia*. The total should be corrected soon as possible.

In myopia this contraction would cause the rays to focus forward still farther, producing *false myopia*.

This condition is designated as *tonic* and *clonic* spasm. The latter is rare.

Tonic Spasm.—The ciliary muscle may remain in a constant state of excessive contraction, making an emmetropic eye hypermetropic. A hypermetropic eye less hypermetropic. And a myopic eye more myopic.

Clonic Spasm.—The ciliary muscle may contract spasmodically when the patient looks at objects. Common with young patients.

The ciliary muscle often contracts and relaxes as lenses of different strength are placed before the eye. Seldom can the proper correction be found unless the ciliary is relaxed.

231. Causes of Spasm.—Continuous work under faulty conditions, or a neglected error of refraction are attributed as the chief factors producing the spasm.

Symptomatology.—Asthenopia.

232. Treatment.—Correct errors of refraction. Suspension of accommodation for a few days or weeks may be necessary. Rest to the eyes. Attention to patient's general health. Some cases *very obstinate.*

233. Paralysis of Accommodation.—*Loss of power of the ciliary muscle* results in lessened or complete loss of accommodation.

Differentiated from presbyopia by the amplitude not corresponding with the table, section 229.

One or both eyes may be affected.

234. Causes of Paralysis.—It may be due to paralysis of the entire 3rd, nerve or of its branch which supplies the ciliary muscle and circular fibers of the iris. Or the cause may be some debilitating disease, such as diphtheria, typhoid, rheumatism, glaucoma, syphilis, etc. or blows about the eye, carious teeth, atropine, etc.

235. Symptomatology.—Inability to accommodate. Loss of accommodation may be complete or partial, depending upon the cause and its prognosis. If the fault lies with the entire 3rd nerve the accommodation is lost, ptosis, strabismus, etc., ensue.

If but the branch supplying the ciliary is affected the diagnostic symptoms consist of lessened accommodation, dilated pupil. Asthenopia.

236. Treatment.—As the object of treatment is to restore the accommodation, the ciliary muscle should be exercised. Therefore, the weakest plus lens if hypermetropic, that will allow the patient to read, should be prescribed. As the ciliary gradually regains its tone, the lenses should be changed frequently. If the patient was fitted for say, three or six inches farther than thirteen inches, and then instructed to read at the regular distance (thirteen inches), benefit would be derived.

Paralysis due to fevers generally yield readily to treatment. But if the patient's condition is a syphilitic one, at present a cure seldom can be effected. Medical treatment as indicated.

Punctum Remotum (Far Point) Punctum Proximum (Near Point), Range and Amplitude of Accommodation

To arrive at the exact condition of the accommodation *i.e.* as to whether it is adequate, the following tests must be made. Punctum Remotum, Punctum Proximum and Amplitude.

The eyes should be examined separately as they may be found to vary considerably, and the distant correction unsuitable for both eyes at nearby work.

237. Punctum Remotum or Far Point (P. R.).—The greatest distance an eye can distinguish objects such as Snellen's No. 20 type, ciliary muscle passive.

238. Punctum Proximum or Near Point (P. P.)—The shortest distance an eye can distinguish an object such as Snellen's No. 1, type, ciliary muscle in highest state of contraction.

239. Range of Accommodation.—Effort required to change the eye from the far to the near point. The amplitude represents the strength of the ciliary muscle, *e.g.* an eye is made emmetropic, and an object is held at a distance of nine inches before it, to have a well defined image upon the retina, the eye must accommodate 4.50 D. From the foregoing explanation it can be seen how the amplitude can be measured in diopters by comparing the P. P. with the focal length of a lens.

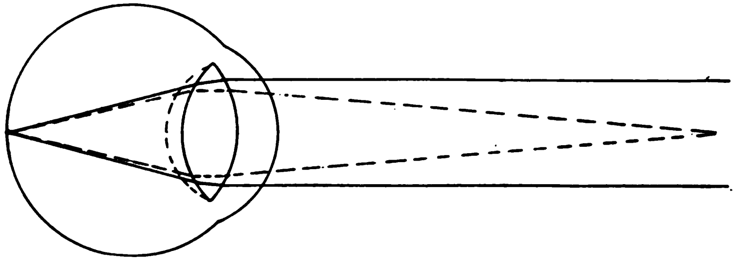


FIG. 129.

240. Terms Defined.—*Absolute accommodation: amount possessed by one eye; the other excluded.*

Binocular Accommodation: amount possessed by both eyes when converging.

Relative Accommodation: amount possessed by both eyes for a given amount of convergence.

Static Accommodation: ciliary in a relaxed state.

Dynamic Accommodation: ciliary in an unrelaxed state.

241. Accommodation in Emmetropia.—In emmetropia, parallel rays focus upon the retina (the ciliary muscle relaxed), because the eyeball is of proper length, and the crystalline lens of proper curvature. Fig. 129. But if the object were at less than infinity (20 ft.) rays from it would strike the cornea so divergent that they would focus back of the retina. To prevent this the ciliary muscle contracts, *i.e.* the eye accommodates, making the lens more convex. See dotted lines from nearby objects and dotted lens.

242. The following table gives the approximate amount of accommodation an emmetrope must put into force for objects at various distances, viz: 20 ft. 80 in., 40., 20 in. etc.

Distance of objects	Amplitude of Acc. in diopters
20 ft. or more	.0
80 in.	0.50
40 in.	1.00
20 in.	2.00
13 in.	3.00
9 in.	4.50
5 in.	8.00
3 in.	13.00

This table is identical with the table "Focal Length of Lenses," Chapter II.

To ascertain the amount a hypermetrope must accommodate for the distances given in the foregoing table, add the amount of hypermetropia to the dioptric number opposite, *e.g.* hypermetropia 3.00 D. To see clearly at 20 inches:

PROCESS:

300 D. (+) 2.00 D. = 5.00 D. *Ans.*

If myopic we, subtract, *e.g.* Myopia —1.00 D. Distance 20 in.

PROCESS:

2.00 D. (—) 1.00 D. = 1,00 D. *Ans.*

In emmetropia, the proper relationship between accommodation and convergence exists because with each degree of convergence one diopter of accommodation is put into force. Thus, if an object is at a distance of one meter (39.37 in. commonly mentioned as 40 inches) to see it clearly and obtain binocular vision, each eye accommodates 1.00 D. and the visual axes form a one meter angle.

The hypermetrope must accommodate more than the emmetrope, and the myope less. Thus it can be seen how in ametropia, there does not exist the proper relationship between accommodation and convergence. Were it not possible to disassociate the two functions, hypermetropes would have divergent squint and myopes convergent squint.

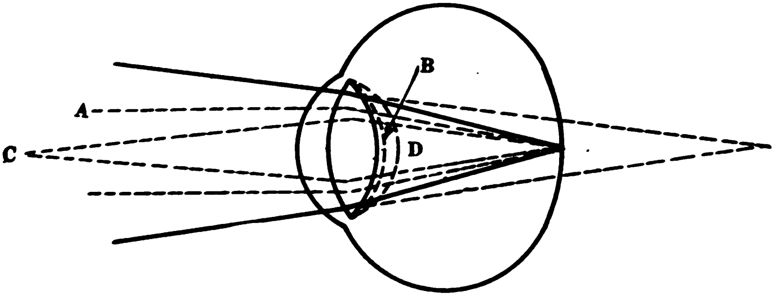


FIG. 130.

243. Accommodation in Hypermetropia.—Parallel rays focus back of the hypermetrope's retina (ciliary muscle relaxed) because the eyeball is too short. Consequently to see clearly at a distance *A*, the hypermetrope must accommodate an amount equal to his refractive error, *e.g.* the hypermetropia was 3.00 D. The hypermetrope must therefore accommodate 3.00 D. for infinity. See dotted circle *B*. Fig. 130,

To cause divergent rays from a nearby object to focus upon the retina, the hypermetrope must accommodate more than the emmetrope, because it is necessary to also overcome the error for distance. Fig. 130. See dotted lines from nearby object *C*, and dotted lens *D*.

244. Accommodation in Myopia.—Parallel rays *A*, focus in front of the retina at *B* (ciliary muscle at rest), because the eyeball is too long. Increased curvature of the crystalline lens would cause the rays to focus forward still farther, making the vision worse. Thus, the myope is helpless to accommodate for distant objects. Fig. 120. Page 103.

If the myopia was 3.00 D. rays proceeding from an object at 13 inches would strike the cornea so divergent that they would focus upon the retina, acc. at rest. Thirteen inches is therefore the punctum remotum, far point. If the distance were at less than thirteen inches, the rays would focus back of the retina. Distinct vision would necessitate accommodation. If the myopia exceeded 3.00 D. rays from any point beyond 13 inches would focus in front of the retina, and to see clearly it would be necessary for the myope to hold the work closer than 13 inches, or wear concave glasses.

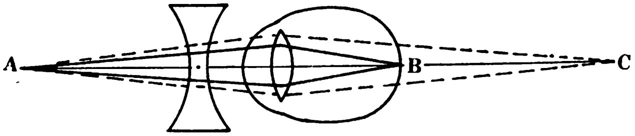


FIG. 131.

Fig. 131. Assume that the myopia, is 3.00 D. Also A, is at a distance of 13 inches. Rays from A, will focus at B. If the distant correction of -3.00 D., is being worn the rays will enter the eye more divergent and thus focus at C. To avoid this the myope must accommodate 3.00 D. In this way the focus is brought to B. Thus distinct vision is obtained.

Through lack of exercise the amplitude of accommodation is frequently found to be diminished.

From the foregoing it is evident that the hypermetrope must accommodate more than the emmetrope, and the myope still less.

245. Accommodation in Presbyopia.—In presbyopia as a result of natural hardening of the lens, the amplitude of accommodation has diminished to such an extent, that an eye cannot accommodate to nine inches. Usually at the age of forty an emmetrope must either wear a plus lens for reading or hold it farther away.

246. Accommodation in Anisometropia.—If the refraction of the two eyes differ but slightly they probably accommodate equally.

If the difference is very great, it is possible that the accommodation is disassociated, and there is also a difference in the amplitude of both eyes.

247. Accommodation in Aphakia.—Removal of the crystalline lens destroys the accommodation. An artificial accommodation can be obtained by resting the lenses upon the nose at different distances from the eyes.

CHAPTER VI

CONVERGENCE

CONVERGENCE DEFINED. METRICAL ANGLE. PUNCTUM REMOTUM. RANGE AND AMPLITUDE. TESTS FOR ORTHOPHORIA. NORMAL RANGE OF MOVEMENT. CAUSE OF MUSCULAR IMBALANCE. HETEROPHORIA AND HETEROTROPIA. DEVIATIONS DESIGNATED. TEST WITH PRISMS, PERIMETER AND MADDOX ROD. TREATMENT.

248. Convergence: Turning Inward of the Eyeballs.—Accommodation is for the purpose of focusing the images of objects situated at varying distances upon the retina. Convergence is for the purpose of so turning the two eyes, that these images will fall upon the macula of each eye, binocular vision being the result. Fig. 132. Diplopia results when the eyes do not converge equally, *i.e.*, the images are not formed upon corresponding points of each retina. Figs. 142, 145.

The eyes are turned inward through contraction of the internal recti muscles.

The amount of convergence exceeds that of accommodation at the far and near points.

249. Metrical Angle of Convergence.—When the eyes are converged to a point in the median line and at less than infinity the visual lines form an angle. If the object is at one meter the angle is called a one meter angle, the unit. If located at one-half meter, a two meter angle. If located at one-fourth meter, a four meter angle, etc. If the object was at say two meters, the angle would be a half meter angle, etc.

If the center of rotation of the eyeballs is 64 mm. apart and the eyes converge to a point one meter away, the visual axis and median line form an angle of 1° 50′. The size of the angle of convergence depends upon the separation of the eyes. The greater the separation the greater must be the effort to converge for a given distance. H should have a reserve equal to about twice that amount required fo his closest working distance.

The approximate angle of convergence can be ascertained a follows:

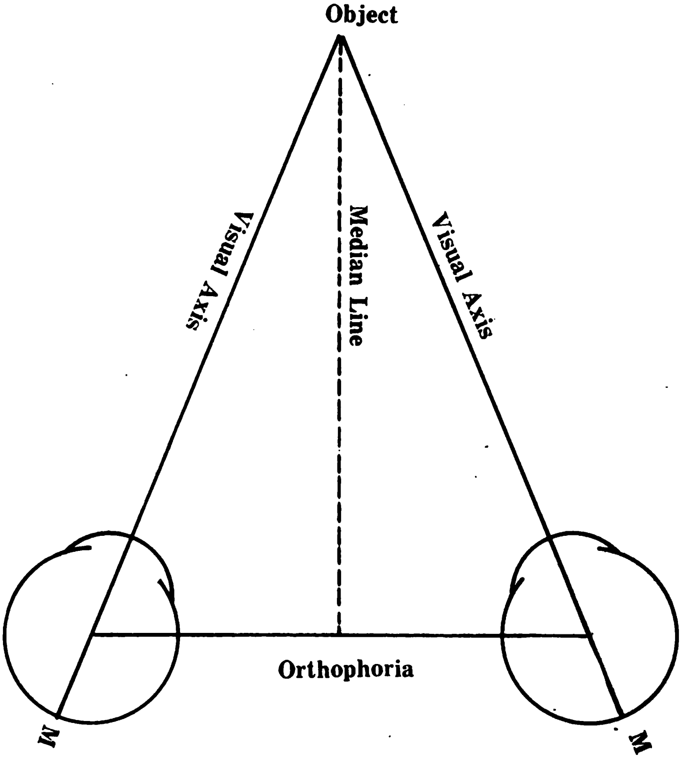


FIG. 132.

Divide 40 by the distance in inches.
EXAMPLE: Eyes converged to point at 20 inches.
SOLUTION: $40 \div 20 = 2$ meter angles. *Ans.*
EXAMPLE: Eyes converged to point at 160 inches.
SOLUTION: $\frac{40}{160} = \frac{1}{4}$ meter angle. *Ans.*

Punctum Remotum. Punctum Proximum. Range and Amplitude

250. **Punctum Remotum (P. R.) of Convergence.**—If when looking into space the visual lines diverge the P. R. is designated (−). If parallel P. R. = ∝. If convergent the P. R. is (+).

Fig. 147. The size of the angle Alpha (angle *a*) depends upon the nature of the ametropia. In emmetropia it is about 5°. In hypermetropia it is much as 8°. In myopia the angle is less than in emmetropia. In a high degree of myopia the optic axis lies inside of the visual axis. The angle Alpha is then designated (−).

251. **Punctum Proximum (P. P.) of Convergence.**—It is the shortest distance to which both eyes are able to converge without producing diplopia.

252. **Range of Convergence.**—Distance between the P. R. and P. P.

253. **Amplitude of Convergence.**—Power required to change the eyes from the far point to the near point. It is conveniently expressed in prism diopters.

254. **Test for Punctum Proximum.**—Place before the eye prism base out. The strongest which does not produce diplopia is a measure in degrees of the punctum proximum.

If the near point is considerably less than thirteen inches, he should be able to converge continuously for thirteen inches, which is the usual reading distance.

255. **Test for Punctum Remotum.**—Have the patient look at a distant object. Prisms are then placed before the eye "base in." The strongest prism that does not produce diplopia represents the negative convergence.

256. **Orthophoria Muscular Balance.**—The ocular muscles are of proper balance, consequently there is no tendency for an eye to deviate. Both eyes converge equally. Fig. 132 illustrates this condition. Lines drawn from the macula of each eye meet at the object when it is at any distance between the far and near points.

Binocular vision is the result. Normal range of movement of the eyeballs is stated as follows: Inward approx. 45°, outward approx. 45° to 50°. Upward approx. 35° to 40°. Downward approx. 60°.

Diplopia may be produced in orthophoria by displacement of the visual axis of one eye as follows: Press the eyeball with your finger, when an object will appear double. Or a prism of sufficient strength will accomplish this.

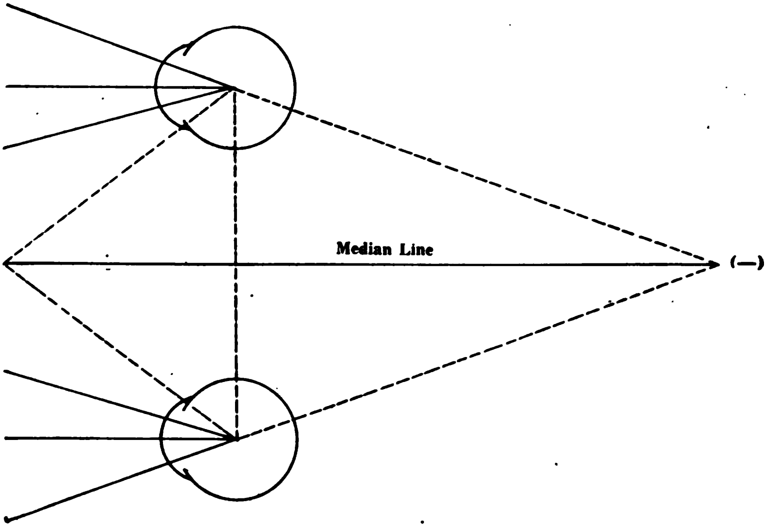


Fig. 133.

257. Heterophoria and Heterotropia; Muscular Imbalance, Cause Of.—In this condition the ocular muscles are not properly balanced, therefore there is a tendency for one or both eyes to deviate in some direction, according to the muscular defect. Lines drawn from the macula of each eye do not meet on the object. Likewise rays from the object do not focus upon the macula of each eye. The two images being formed upon non-corresponding points the patient sees double. If the tendency to deviate is slight the patient may overcome this error. The cause may be of a muscular or nervous origin; a refractive error; opacity of the cornea or lens; or defective vision in one eye.

Muscular imbalance may result from the disturbed relationship between accommodation and convergence as follows:

The nerve innervation of both the muscles of accommodation and convergence are supplied by the same branch of the third nerve. If both muscles receive the same amount of stimulus in emmetropia binocular vision results, provided the other ocular muscles are of normal tone. But in a high degree of hypermetropia, excessive accommodation being necessary, if the internal rectus muscles received the same amount of innervation as the ciliary the eyeballs would turn inward too far, causing esophoria.

In a high degree of myopia there is no demand on the ciliary muscle, consequently the nerve supply of the converging muscles is apt to be insufficient. Added to this, the extra effort necessary to rotate the long myopic eyeball there is a tendency toward exophoria, Fig. 135.

About 80 per cent. of the cases of esophoria occur in hypermetropia. In exophoria, the patient is usually myopic.

Most persons can disassociate the accommodation and convergence. Were it not for this, muscular imbalance, according to the foregoing, would invariably accompany ametropia.

258. Deviations Designated.—An eye deviates away from an affected muscle. Thus if say the internal recti muscles are affected, the eyes will deviate outward.

If an eye deviates inward, deviation designated esophora, Fig. 134. If outward, exophoria, Fig. 135. If upward, hyperphoria, Fig. 136. Upward and out, hyper-exophoria, Fig. 137. Up and in, hyper-esophoria, Fig. 138. Downward, cataphoria, Fig. 139. Down and in, eso-cataphoria, Fig. 140. Down and out, exo-cataphoria, Fig. 141.

If the tendency to deviate is slight it is termed heterophoria. If the deviation is considerable it is designated heterotropia, strabismus, diplopia, squint and "cross eye."

FIG. 134.

FIG. 135.

FIG. 136.

FIG. 137.

FIG. 138.

FIG. 139.

FIG. 140.

FIG. 141.

Heterophoria. Insufficiency. Tendency to Deviate. Slight Deviation

259. **Heterophoria.**—Visual axes parallel for distant vision but deviate when the extrinsic muscles relax.

If both images appear close together as in heterophoria, the *false* is almost as distinct as the *true* and causes much discomfort. If far apart the false is indistinct, because the visual acuity diminishes rapidly as the periphery of the retina is approached. The brain may ignore the false image.

Heterotropia. Squint. "Cross Eye" Strabismus

260. **Squint** is classified as follows: concomitant, alternate and paralytic. If one eye is affected the squint is termed *monolateral.* If both eyes, but alternating, it is termed *alternating.*

261. **Apparent Squint.**—When the visual axes are divergent in hypermetropia and convergent in myopia the eyes have the appearance of squint.

262. **Heterotropia.**—Deviation of the visual axes or axis so marked as to be observable, deviations designated as follows: If inward, esotropia, convergent strabismus. If outward, divergent strabismus. If upward, strabismus sursumvergens. If downward, strabismus deorsumvergens.

Homonymous and Heteronymous Diplopia

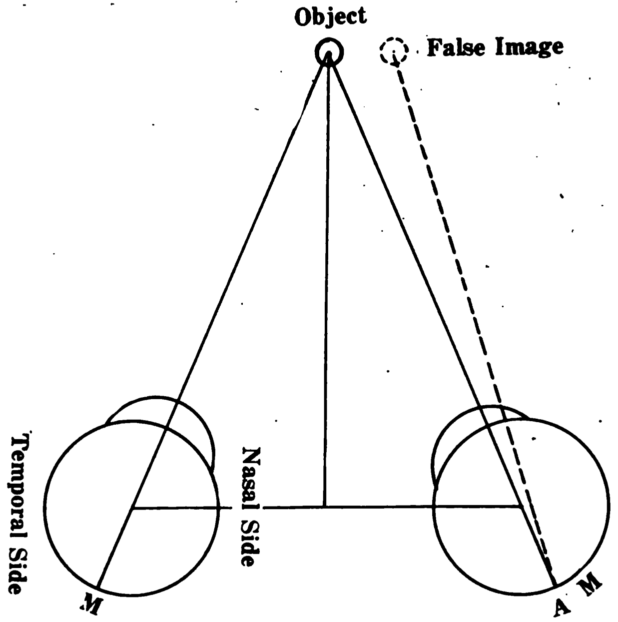
263. **Homonymous Diplopia.**—A clear understanding of the location of the false image can be obtained from the following explanation, Figs. 142, 143.

A line from the object falls upon the nasal side of the macula. The image is projected outward toward the temporal side.

264. **Heteronymous Diplopia.**—Fig. 144. A line from the object falls upon the temporal side of the macula. The image is projected inward toward the nasal side.

If this line falls at a point above the macula it is projected downward. If at a point below the macula it is projected upward.

265. Heterotropia, Heteronymous Diplopia, Exophoria.—Fig. 144. The left eye converges properly and rays from the object focus upon the macula. This is *the true* image. The right eye deviates outward, *i.e.*, it does not converge the same amount as the left eye.



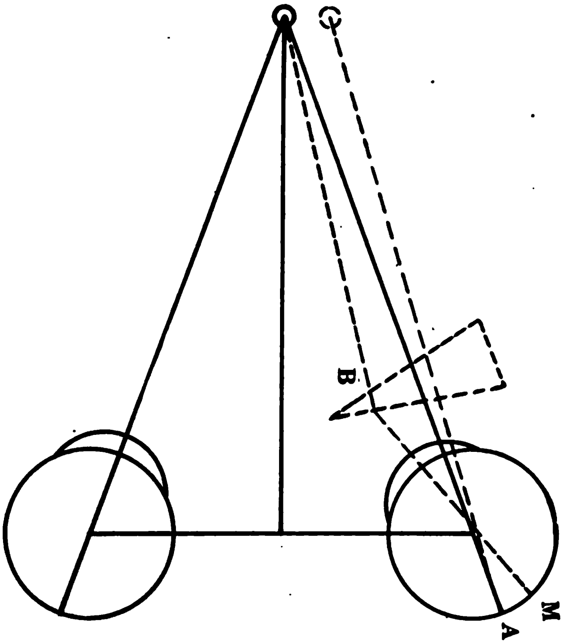
Homonymous, Diplopia Right image belongs to right eye Left Image belongs to left eye

FIG. 142.

Consequently rays from the object do not focus upon the macula, but at *A*. This image is *false* and fainter. The images being formed upon non-corresponding parts of both retinas the brain is unable to fuse them. Diplopia (double vision) results.

In diplopia objects appear double. In strabismus as a rule the brain ignores the false image. Where one eye deviates outward, the diplopia is termed *crossed diplopia.*

If a prism of proper strength, base in, is placed before the deviating eye, Fig. 145, a set of rays from the object would enter it at say *B*, and being refracted toward the base would after passing through it follow the visual axis and focus upon the macula, thus producing binocular vision.



Homonymous, Diplopia Right image belongs to right eye Left Image belongs to left eye

FIG. 143.

266. Muscular Imbalance. Esophoria.—Fig. 143. Eye deviating inward. The left eye fixes the object, the right eye turns or deviates inward too far. Rays from the object focus upon the nasal side of the macula at *A*. If a prism of correct strength is placed before the eye a set of rays would enter it at *B*, and being refracted toward the base they would focus upon the macula. The images not being formed upon corresponding "identical" points of the two retinas; the brain does not fuse them and the patient sees "double." Image

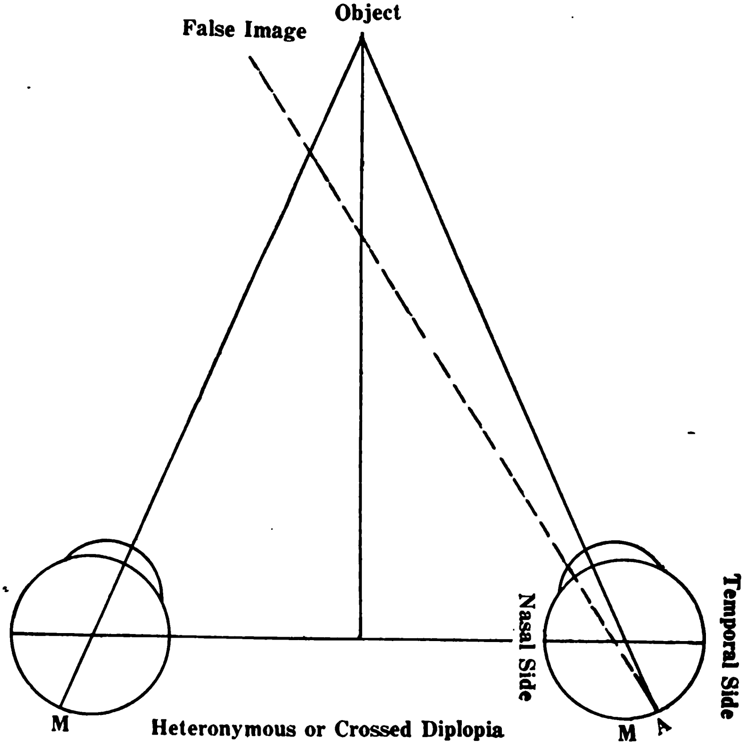
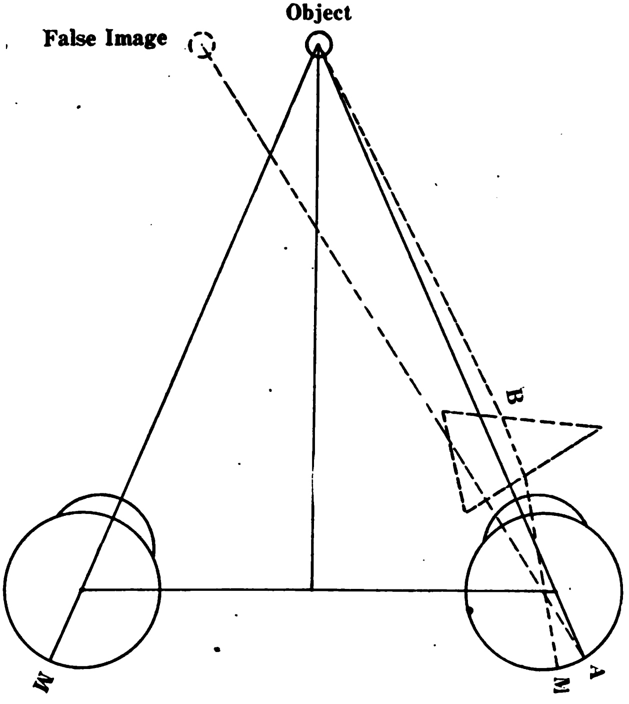


FIG. 144.

being formed on inner side of macula. Diplopia termed "homonymous diplopia."

If the patient has a weakened superior oblique the images may be arranged as in Fig. 146. As in looking downward during the act of convergence the superior oblique lags and being out of balance with



Heteronymous or Crossed Diplopia

FIG. 145.

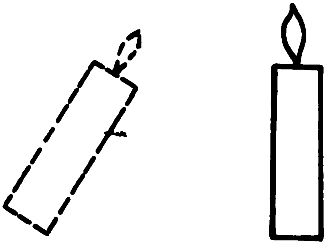
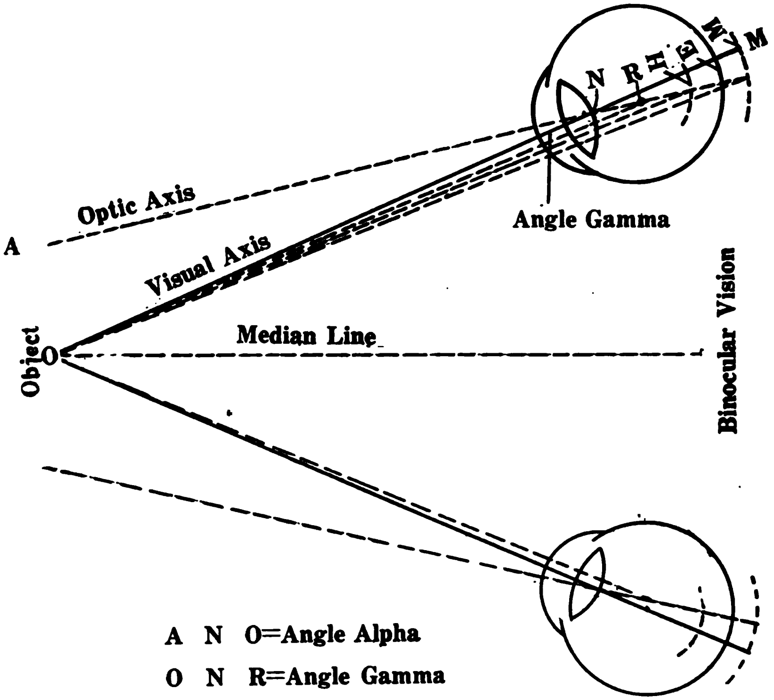


FIG. 146

the inferior oblique, a wheel-like rotation of the eyeball takes place. Hence the slanting image.

Fig. 147 explains why the myopic eye on account of its length makes a wider movement when converging for a given distance than the emmetropic or hypermetropic eye.



A N O=Angle Alpha

O N R=Angle Gamma

FIG. 147.

This distance is represented by the distance between M and O. Thus the angle Alpha is smaller in myopia. On account of this great distance the myopic eyes often appear convergent, Fig. 134. On account of the short distance the hypermetropic eyes often appear divergent, Fig. 135.

The approximate angle Alpha in emmetropia is about 5°.

In hypermetropia 7° or 8°.

In myopia 2°; sometimes negative.

267. **Nystagmus.**—Described as spasmodic, involuntary oscillation of the eyeballs.

268. **Causes.**—Usually an infantile disease, and occurs where vision is impaired due to opacities of the refractive media. A few cases have been discovered among coal miners. Accommodation and emotion disturb the condition.

269. **Symptoms.**—Movement usually horizontal and from right to left. Sometimes movement in vertical plane. Seldom in a diagonal plane. These conditions are *oscillatory nystagmus. Rotary nystagmus* is a condition in which the eyeballs describe a rotary movement around the optic axis. Usually a difference in the vision of both eyes.

Medical treatment does not come within the scope of this treatise.

270. **Causes of Muscular Imbalance.**—The causes may be of a muscular or nervous origin. Refractive error. Opacity of the cornea or lens. Defective vision in one eye.

271. **Paralysis of the Third Nerve.**—Paralysis if complete, drooping upper lid (ptosis). Anterior end of ball drawn outward and slightly downward by external rectus and superior oblique muscles. Lost accommodation. Dilated pupil. Crossed diplopia. If the paralysis is partial the above symptoms are modified. Some may be absent. If the branch that supplies the internal rectus is paralysed, convergence is affected. Crossed diplopia. Images are parallel and occupy the same horizontal plane.

If paralysis affects the inferior rectus, downward and inward rotation of the ball is affected. Diplopia is under horizontal plane. Crossed images. False below true and images converge at top.

272. **Paralysis of Fourth Nerve.**—Superior oblique affected. Downward and outward motion of ball impaired. Homonymous diplopia below horizontal plane. False image located below true and they converge at top.

273. **Paralysis of Sixth Nerve.**—External rectus affected. Impaired outward rotation of ball. Homonymous diplopia. Images parallel and on same horizontal plane.

The patient looking at a small light, situated at a distance, sees with the eye over which the Maddox Rod is placed, a narrow "band" or "streak" of light. This streak is at right angles to the meridian occupied by the rod.

The other eye sees the flame. Unless the band extends through the flame, Figs. 154 and 155, the muscles under test are not of proper balance. The visual axes are not parallel, *i.e.*, the punctum remotum (far point) of convergence is not at infinity.

The greater the separation of "band" and flame, the greater is the muscular imbalance. Prisms are to be placed before the eye, until the band is caused to extend through the flame, which is termed *fusion*. The prism which produces fusion is spoken of as the neutralizing prism. It is a measure of the muscular imbalance in prism degrees.

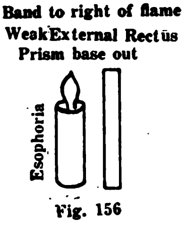


FIG. 156.

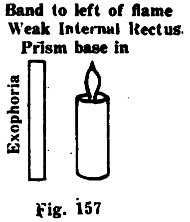


FIG. 157.

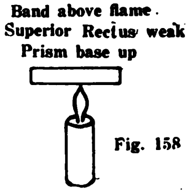


FIG. 158.

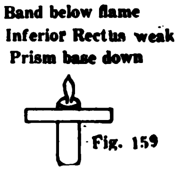


FIG. 159.

279. Procedure.—Seat the patient six meters (20 feet) from a small light. A candle or screened light are satisfactory. Carefully adjust the trial frame for pupillary distance. Place in the frame the patient's distant correction. Next place the Maddox Rod before one eye. We will assume the right eye. Adjust the glass rod so that it is horizontal (180th meridian). The eye over which the rod is placed will see a vertical "streak" or "band" of light. The other eye will see the flame. Unless the band extends through the flame, Fig. 154, the internal and external recti are not properly balanced. Should the band appear to the right of the flame, Fig. 156, the external recti are insufficient. Esophoria homonymous diplopia.

Place before the eye prisms with their bases outward, Fig. 148, until fusion takes place. Record the strength of the prism.

If the band appears to the left of the flame, Fig. 157, the internal recti are insufficient. Exophoria heteronymous diplopia. Place before the eye prisms base in, Fig. 149, until fusion takes place. Record the strength of the prism.

To test the superior and inferior recti, place the rod vertically over one eye. We will assume the right. Unless the band extends through the flame, Fig, 155, the muscles under test are improperly balanced.

If the band appears above the flame, Fig. 158, the superior rectus is at fault. Cataphoria. Place before the eye prisms, base up, Fig. 151, until fusion takes place.

Should the band appear below the flame, Fig. 159, the inferior rectus is at fault. Hyperphoria. Place before the eye prisms, base down, Fig. 150, until fusion occurs. Next examine the left eye.

280. Prisms are combined with spherical and spherocylindrical combinations. The neutralizing prism may be ground in one lens or its strength divided between the two. Or the lenses may be ordered decentered. On account of excessive weight, chromatic aberration and unsightly appearance prisms of more than 5° are seldom prescribed.

281. Test for Squint.—Hold your finger about three feet away from the patient's eyes and have him look at it. Then advance the finger. If the squint is simply *apparent* both eyes will follow it, converging equally. Either eye can fix the object but the two cannot in unison. If the squint is *paralytic* the paralyzed eye may or may not follow the advancing finger for a distance. That would depend upon the degree of the paralysis. A fluctuating movement of the affected eye will be observed shortly before it reaches its maximum travel. *In alternate* squint either eye can fix the object. Cover one eye; a ground glass disc is excellent; have the other fix the object; suddenly remove the disc. If the eye turns inward it is the deviating eye. If the unaffected eye is covered the rotation that it makes to fix the object is spoken of as the *secondary deviation.*

282. Another Test.—Circle an object before the eyes.

In *concomitant* both eyes have an equal range of movement. In *paralytic* the range of movement is limited toward the paralyzed side.

283. Measuring Strabismus with the Perimeter.—The arrangement, Fig. 160, is similar to that when ascertaining the field of vision. The deviating eye is in the center of the arc and in the path of a line drawn from some small object (*A*) across the room and through the (*O*) mark on the arc. A small candle makes a very suitable object. He is instructed to fix the candle with the other eye. The refractionist then slides another candle (*B*) around the arc until the flame appears to be directly in front of the deviating eye. A measure of the strabismus is indicated by the graduation on the arc.

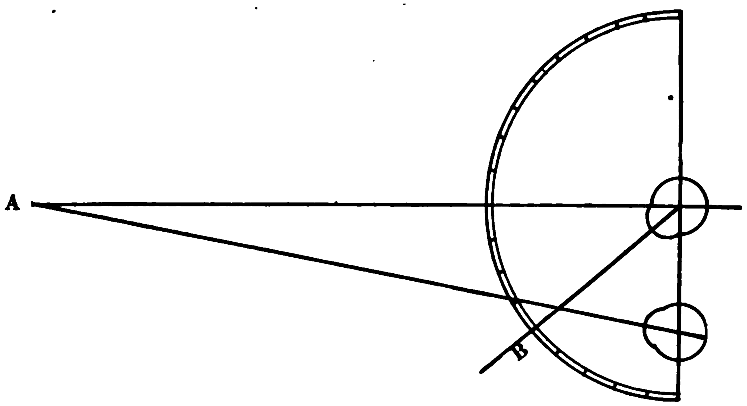


Fig. 160.

284. Treatment for Squint.—Muscular imbalance will disappear in about 80 per cent. of all cases if the ametropia is corrected. It is advisable to correct the latent ametropia soon as possible. In strabismus as a last resort tenotomy is sometimes performed. Operations are not discussed in this treatise. Much precaution should be observed when prescribing prisms as the amount of deviation will sometimes vary at different sittings.

285. Exercise Prisms.—It is better to not overtax the convergence at first. Place in the trial frame prisms that do not quite equal the patient's amplitude. Have him look at an object across the room for a few minutes, lifting the frames at intervals, for a few seconds.

An excellent plan is to prescribe a pair of prisms equal to about one half of his amplitude. These are to be worn over the lenses at his work for a half hour or so daily. As the internal recti increase in strength these are to be exchanged for stronger ones. In some cases the strength of the recti muscles can be doubled.

286. Decentering Lenses, to Obtain a Given Prismatic Effect.—RULE A. 1.00 D. lens decentered 10 mm. produces a prismatic effect of (1°) one degree.

These problems may be treated by the following simple formula. Multiply the prism in the prescription by 10 and divide by the dioptric number of the lens.

EXAMPLE.—Prescription +5.00 D sphere $\bigcirc$ 2° prism. Solution—

$\frac{2 \times 10}{5}$ = 4 mm. decentration.

When cylinders are to be decentered it must be remembered that no prismatic effect will be produced in line with the axis. But at right angles to its axis the prismatic effect is the same as sphericals.

Lenses ground decentered are known as prismospheres.

287. Decentration of Convex and Concave Lenses.—A convex lens may be looked upon as a number of prisms arranged in a circle with their bases placed together. Concave lens may be looked upon as a number of prisms arranged in a circle with the apices placed together and bases outward.

288. Convex Lenses.—If it is desired to obtain the effect of a prism, base in, to aid convergence, crowd the lenses together so that their optical centers are between the pupils. If it is desired to obtain the effect of a prism base out, spread the lenses so that their optical centers are outside of the pupils.

289. Concave lenses are decentered opposite to that of concave lenses, *i.e.*, the optic centers come outside of the pupils.

From the rule just given, it is plain that in order to obtain a given prismatic effect, a weak lens would require considerable more decentration than a strong lens. Also that if the ametropia was of a small measure, and the muscular insufficiency considerable, that the lenses could not be decentered a sufficient amount. It would be necessary to specify a prism in the prescription.

In fitting frames it should be borne in mind that unless the centers of the lenses are exactly in front of the pupils they will act as prisms, the tendency being to displace the visual lines. In a high degree of ametropia the prismatic power of the lenses is correspondingly great and inaccurately centered lenses would in many cases cause troublesome asthenopia.

290. **Asthenopia.**—Discomfort, which when accompanied by use of the eyes is referred to as asthenopia.

Asthenopia is classified as follows: *Muscular, accommodative and reflex.*

291. **Muscular Asthenopia.**—Due to an insufficiency of some of the recti muscles. When present, the case is usually one of high myopia. Symptoms more pronounced on close work. The letters "dance," there being a tendency toward diplopia.

292. **Accommodative Asthenopia.**—Due to insufficiency of the ciliary muscle. Accommodative is usually met with in hypermetropia. In close fine work the symptoms manifest themselves. Work appears dim or foggy, but clears up when the eyes are rested for a few minutes.

293. **Reflex Asthenopia.**—Due to a hypersensitive eye. Causes similar to those in neurasthenia.

294. **Symptomatology.**—Eyes feel heated, heavy, tired, and painful. Pain in head and back of neck. Drowsiness, vertigo, nausea, and vomiting. Symptoms may occur while the eyes are being used or several hours later.

295. **Causes.**—Usually the cause can be traced to either an error of refraction, weakness of an occular muscle or ill health.

If the cause is a reflex one, we may suspect some organic disturbance. In the case of young unmarried women a weakness peculiar to their sex; young men may have violated the moral laws. Retinal fatigue, common with gold workers and seamstresses. Carious teeth, etc.

296. Treatment.—Correct the total ametropia soon as possible. If there is a weakness of any ocular muscle, it should receive careful attention. Rest when symptoms remain. Medical treatment as indicated.

CHAPTER VII

CYCLOPEGIC

297. The purpose of the cycloplegic is to temporarily paralyze the ciliary muscle. The accommodation is then said to be "relaxed," "suspended," "at rest," "passive," "static." With the ciliary muscle thoroughly relaxed, the exact optical condition of the eye can be ascertained. Thus in emmetropia, parallel rays (rays which have proceeded from an object situated at a distance of twenty feet or farther), will focus upon the retina. Therefore, in emmetropia, lenses for distant vision are unnecessary. In fact a plus lens would cause the rays to focus in front of the retina as in myopia. A minus lens would cause them to focus back of the retina as in hypermetropia. In the hypermetropic eye, parallel rays focus back of the retina. It requires a plus lens of proper strength to cause them to focus upon the retina. In myopia, parallel rays focus in front of the retina. A minus lens of proper strength is required to cause them to focus upon the retina.

In the case of children and young adults the crystalline lens is very flexible and as a rule, the ciliary muscle is of abnormal tone. Consequently, to find a lens which will be satisfactory is often impossible until after the ciliary has been thoroughly relaxed. If the cyclopegic has been administered latent hypermetropia if present, can be measured, so also can acquired myopia and false astigmatism.

The cycloplegic is administered with the hope that latent hypermetropia will become permanently manifest, and acquired myopia and false astigmatism will disappear permanently, as there is very apt to be asthenopia and more or less trouble with the lenses (in many cases frequent changes) until this condition is brought about. In some cases it is only necessary to keep the patient atropinized for a day. In other cases children especially, two weeks is inadequate, the ciliary cramp returning as the cycloplegia passes off. However, the rest to the eyes is usually of much benefit. It is fortunate that usually before the age of thirty latent and acquired ametropia have disappeared.

When under the influence of a cycloplegic, a hypermetropic eye will be about .50 D. more hypermetropic and a myopic eye about .50 D. less myopic. This ametropia is false. At this period a record should be made. The test for the lens to be worn is to be made from five to seven days after the last installation of atropine.

Hom-atropine produces a condition of paralysis quickly and the effect passes off in about twenty four hours. The test should be made in a few hours after the last installation. If it is necessary to make the test while under the influence, subtract .50 D. if the patient is hypermetropic, and add .50 D. if myopic. This treatment is popular with adults.

Formulas.—Atropine sulphate, 4 gr. to an ounce of distilled water. Instill in outer canthus of eye. Produces rapid dilitation. If instilled four or five times daily for three or four days, it will relax the accommodation.

Hom-atropine, 2½ per cent. solution. Instill at intervals of five minutes for six times. Acts as a thorough cycloplegic in from one to three hours. C. M. Culver, Anals of Ophthalmology, January, 1900.

298. Precautions.—Atropine should not be administered when symptoms of glancoma are present.

It should not be administered to nursing mothers.

In heart disease the toxic effect should be considered.

Some few have an idiosyncracy for the drug.

299. Mydriatic.—The purpose of a mydriatic is to temporarily dilate the pupil. The pupil becomes larger when the sphincter muscle is relaxed. A mydriatic is sometimes administered in making the ophthalmoscopic examination and in Retinoscopy. Also surgical operations.

Atropine will act as a cycloplegic and mydratic Cocain dilates the pupil but does not paralyze the ciliary muscle.

CHAPTER VIII

VISUAL ACUITY AND SUBJECTIVE TEST

300. Field of Vision and Colors.—The field of vision both monocular (one eye being examined), and binocular (both eyes being examined together), should be tested to ascertain if it is normal of limited. A record should be preserved for future reference.

A condition of limitation may arise from causes, congenital or acquired.

In cases where the vision cannot be brought to normal by correcting lenses the trouble is often found to lie with the optic nerve fibers or fusion centers. Early treatment as indicated.

Monocular Field of Vision.—Is the area over which an eye sees when in a fixed position on looking at a point.

Binocular Field of Vision.—Is the area of the two fields where they overlap, when the eyes are directed at an object situated in the median line.

301. Test for Field of Vision and Colors.—The most accurate test is with the perimeter, Fig. 161. Charts of both the field of vision and colors, of a normal eye may be obtained from the optical houses.

Following fs a fairly accurate method.—Seat the patient comfortably, about a foot from a well illuminated black board. Cover the eye not being examined. Make a small mark on the board, directly in front of the patient's eye. The patient is instructed to fix his gaze upon this point. Fasten a small piece of chalk or a white object to a rod. Then place the object against the board, some distance from the fixation point, and move it toward this point. The patient is instructed to mention the fact when it first comes into view, when a small mark is made. Continue moving the object until the fixation point is reached. Different places on this line where the patient says that the object is either indistinct or invisible, correspond to parts on the retina where the sight is impaired or destroyed. Move the object in the different meridians.

Through the dots that indicate where the object first came into view (not distinguished as an object) draw a line. In this way a map of the *quantitative field* is obtained.

Connect the dots where the object was first distinguished. The enclosed area is the *qualitative field.*

Where the inner half of each field is lacking the condition is termed, *Nasal Hemiopia.* If the outer fields, *Temporal Hemiopia.* If the right or left fields, *Homonymous* or *Equilateral Hemiopia.*

The location of the blind spot on the optic disc is to be ignored. Its location is not the same in all eyes.

By the arrangement and process previously described the field of different colors can be tested. Use a colored object corresponding with the color being tested for.

303. Test for Field of Colors.—Limitation may be congenital or acquired.

The worsted test is considered a rigid one. First test each eye separately and then both together. Select a skein of the primary colors, say red and its various shades. The patient is instructed to select the red piece and then place beside it all samples that he judges to be of the same color and matched according to shade.

FIG. 161.

The colors green, yellow, etc., should also be used.

The examiner should carefully note if the patient is looking directly at the object, termed *central fixation,* the image being formed upon the macula or sideways, *eccentric fixation,* the image not being formed upon the macula. Examine each eye separately and then both together. Prof. Holmgren's test.

303. The Trial Case.—As accuracy is such an important factor in refraction work, the tools should be high grade in every respect.

The trial case, Fig. 162, should contain at least the following: Plus and minus spherical lenses from .12 D. to about 20.00 D. Cylindrical lenses from .12 D. to about 8.00 D. Prisms ranging from 1° to 25°. Muscle Test, Stenopaic Disc, Opaque Disc, Pin Hole Disc, Retinoscope, Ophthalmoscope and Adjustable Trial Frame, 163.

FIG. 162.

The trial frame should have both horizontal and vertical adjustments, so that when a trial lens is placed in the cell, it will occupy the same position as when in the frame prescribed for wear.

304. Visual Acuity and Test Types.—The distance at which an eye can distinguish an object of given size is taken as an index of the visual acuity, Snellen's test type is based upon the principle that each part of a letter is separated by an interval of one minute (1′) and the entire letter subtends an angle of five minutes (5′), Fig. 165. If a patient can distinguish the No. 15 letter at fifteen feet the visual acuity is expressed ($^{15}/_{15}$) normal. But if it cannot be read at fifteen feet, but can at ten feet, vision is expressed $^{10}/_{15}$, *i.e.*, below normal. If the letters cannot be read easily at the required distance, spherical lenses should be used. Then if the vision is not rendered normal, test for astigmatism. In astigmatism, the correcting combination does not always produce normal vision at once, but it usually improves in a few weeks. If it does not, the perimeter may disclose a limitation of the field of vision. Or the ophthalmoscope may reveal a lack of transparency of the media. A simple way to settle this question is with the pin-hole test as follows: An opaque disc with a pin-hole opening made in the center is held accurately before the center of the pupil of the eye under examination, the other eye excluded. If the vision is improved, an error of refraction is indicated, which in all probability can be corrected. But if there is no improvement, the fault lies with the nervous system, or there may be a lack of transparency of the media.

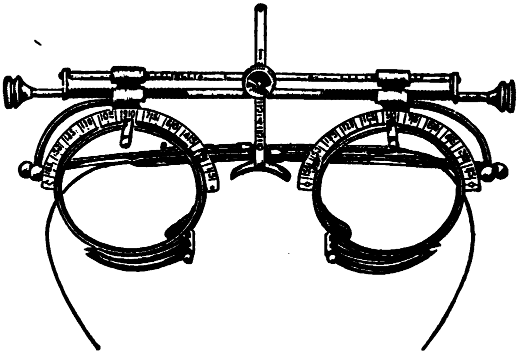


FIG. 163.

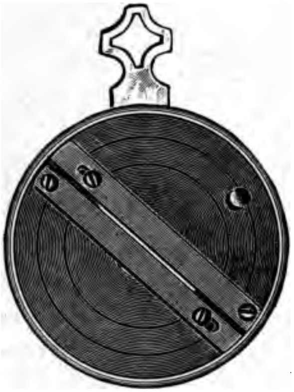


FIG. 164.

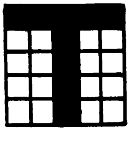


FIG. 165.

In some cases, especially children, the visual acuity is above normal. The No. 15 letter being read at twenty feet. Acuity then expressed $^{20}/_{15}$. It decreases slightly with advancing age.

A record of the visual acuity with and without lenses should be preserved for future reference.

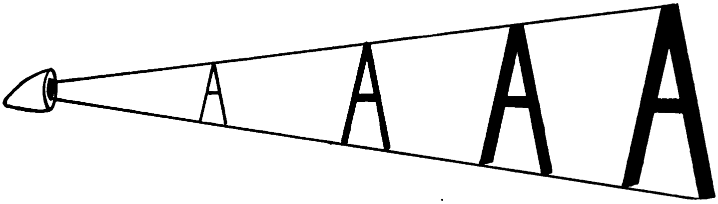


Fig. 166.

Fig. 166 illustrates the letters Nos. 5, 10, 15 and 20 at the respective distances of 5, 10, 15 and 20 feet, subtending the same angle of $5'$. It can be seen that the letters increase in size pro ratio, and if the No. 5 can be read at 5 feet the No. 10 letter can be read at 10 feet, etc. In this case the vision would be expressed 5/5, 10/10, etc. which terms designate normal vision.

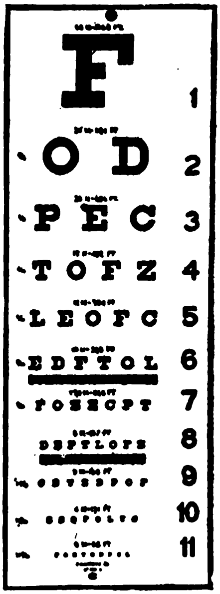


Fig. 167.

Fig. 168.

Fig. 169.

There have been a great many charts devised for testing the visual acuity. Fig. 167 is a specimen of Snellen's type. Fig. 168, the astigmatic ("clock dial") chart. To accommodate the illiterate and foreigners, the ordinary chart would be inadequate. Fig. 169 is a chart for this purpose.

In making the subjective test, the patient is seated at a distance of, we will say, fifteen feet, from the test type. He is then asked to read the letters numbered fifteen. If he cannot, place a plus lens say +.25 D. in the trial frame instructing him to watch the line No. 15. If he is hypermetropic this line (excepting in some cases) will appear more distinct. Continue testing with plus lenses of increasing strength until it can be easily read. The lens which enables him to do this is the correcting lens, and is a measure of the hypermetropia. The procedure in testing a myopic eye is identical, excepting that minus lenses are used.

Subject Test for Simple Hypermetropia, Simple Myopia and Astigmatism

305. Subjective Test.—*The refractionist selects what he considers the proper lens. This selection depends upon how well the patient sees. Therefore the examination is considered subjective.*

In making an examination the following instructions and precautions should be carefully borne in mind.

Seat the patient comfortably at a distance of 5, 10, 15 or 20 feet, from the test type, which should be properly illuminated. It is customary to first examine the right eye.

The trial frame, Fig. 163, should be carefully adjusted, so that the centers of the trial lenses will be opposite the centers of the pupils.

Test each eye separately. Be very careful that the opaque disc, which is to be placed over the eye not being tested completely covers it, and always be on your guard as most patients turn the head sideways to bring into use the covered eye.

After the eyes have been examined separately, try them together at the type. It may be found that a slight change in one or both lenses will be of benefit.

Always commence with plus lenses, as they have a tendency to relax the accommodation, while a minus lens, if not needed, may excite it more or less.

Plus lenses may be placed before the eye from weaker to stronger, or vice versa. But with minus lenses, always commence with the weakest.

Many persons will persistently read the entire chart when a different lens is tried. Much time as well as fatigue to the patient can be saved by directing his attention to the line to be read.

The patient should not be allowed to make a special effort to distinguish the type, as the accommodation may be disturbed. In the subjective test the letters should be read without effort, when aided by the correcting lens.

Some patients, especially children, memorize the test type rapidly. This can be obviated by having several charts with differently arranged letters.

An intoxicated or excited patient is an unfit subjective for examination.

In both hypermetropia and myopia, especially if the ametropia is of a high degree, little or no improvement may be noted until near the correcting lens.

If plus lenses improve the vision, it is a case of hypermetropia.

If neither plus or minus lenses obtain normal vision, test for astigmatism.

In astigmatism, arriving at the proper correction is often difficult and sometimes impossible.

If the ametropia is of a high degree, and the astigmatism slight, it may not be detected until after a partial correction has been made.

In any refractive test the accommodation should be passive.

In looking at an object situated at infinity (which means at a distance of 20 feet or beyond), and the accommodation at rest, parallel rays should focus upon the retina. This ideal condition is found in emmetropia. A plus lens enables the hypermetropic eye to see distinctly at infinity, relieving it of the task of accommodating. A minus lens enables the myopic eye to see well at this distance, where it is helpless to accommodate. Therefore, as an eye should not accommodate for distance prescribe the strongest plus lens in hypermetropia for, if the eye is not fully corrected it will be obliged to accommodate some. Prescribe the weakest minus lens in myopia, for if over corrected the concave lens will cause parallel rays to focus back of the retina, making the eye artificially hypermetropic, instead of emmetropic.

Before ordering the spectacles, at least two examinations should be made and the patient allowed to read with the lenses for at least half an hour. The lenses should be placed in the trial frame so that they will occupy the same position as when in the spectacle frame.

306. Fogging Method.—The patient is rendered myopic, by fogging the vision, with a plus lens of sufficient strength, *i.e.*, a lens through which objects appear hazy. The strength of this lens is then reduced a fraction of a diopter at a time, until best vision is obtained.

It is based upon the principle that any contraction of the ciliary muscle in myopia would result in the vision becoming still more blurred. Therefore, improvement can only be affected through relaxation of the ciliary.

The fogging method can be practiced as follows: First fog the vision with a plus lens. Then place with it another plus lens a fraction of a diopter weaker, at the same time remove the stronger lens. Then replace the lens which is now in the trial frame with one weaker. Repeat this process until the type can be read at the regulation distance.

Another procedure is to fog with a plus lens which is to be left in the frame during the test. Then place in the cell next to it a weak minus lens. Replace this minus with one slightly stronger. Continue this process until the correcting combination is found. If the minus lens is of less refractive power than the plus lens, the case is one of hypermetropia, *e.g.*, the combination was +5.00 D. ◯ −2.00 D. = +3.00 D. The eye was hypermetrpic +3.00 D. If the minus lens was of greater power than the plus, *e.g.*, +5.00 D. ◯ −8.00 D. = −3.00 D. The eye was myopic −3.00 D.

Subjective Test for Simple Hypermetropia

307. Test with Plus Spherical Lenses.—Commence by placing in the trial frame a +.25 D. lens. If the line to be read does not become dimmer, replace the +.25 D. with one a little stronger, say +.50 D. Continue this process of successively placing before the eye plus lenses of increasing strength, until the line to be read is perfectly distinct. Then increase the strength until the line is slightly blurred. Upon repeated trials, the correcting lens will be found among these few lenses.

If no plus lens obtains normal vision, test for simple myopia or astigmatism.

Test for Simple Myopia

308. Test with Minus Spherical Lenses.—Commence with a weak minus lens and place before the eye successively lenses .25 D. stronger. Stop with the weakest through which the patient can see best.

If no minus spherical lens is found satisfactory, test for astigmatism.

309. Test for Regular Astigmatism.—Have the patient look at the clock dial chart, Fig. 168, which is to be placed at a distance of twenty feet or so. Ask him if all lines appear equally plain. If he says that one set is more distinct than those at right angles, regular astigmatism may be present.

An understanding of the following points will aid greatly in making an examination.

The meridian of least ametropia is in line with the dimmest lines on the chart.

The meridian of greatest ametropia is in line with the plainest lines on the chart.

We will assume that the patient sees the vertical lines more distinctly than the horizontal lines. Instruct the patient to observe these vertical lines as you will endeavor to make them still more distinct. Place before the eye say a +.50 D. sphere. If this glass improves the vision, continue with the plus spheres until he states that the lines are growing less distinct, which indicates that an overcorrection has been made. When this occurs, reduce the strength of the plus sphere until best vision is obtained. This lens should be placed in a cell next to the eye so that it will cause no bother when using the cylinders. The patient should now be instructed to observe the lines at right angles, that is, those in the horizontal meridian as you will undertake to bring them out more distinctly. Next place before the sphere a weak minus cylinder with its axis over the lines being observed. Continue with the minus cylinders from weaker to stronger until the horizontal lines are equally plain, *i.e.*, all lines in the chart are of equal clearness.

In the prescription specify the sign and strength of the spherical. Also the sign, strength and axis of the cylinder. This combination will be the patient's distant correction.

Suppose the patient is myopic and he states that the vertical lines are plainest. He is instructed to observe the vertical lines. If you placed say a +.50 D. spherical glass in the trial frame he would exclaim that the vertical lines are still less distinct. You are then to remove the plus glass and commence with a very weak minus spherical increasing the strength a fraction of a diopter at a time, stopping with the weakest that renders the vertical lines plainest. Next tell him that you will undertake the lines at right angles, *i.e.*, the horizontal meridian. These particular lines are to be brought out with minus cylinders placed before the minus sphere, from weaker to stronger, axis horizontal. Stop with the weakest that renders the entire wheel uniform.

In the prescription state the sign and strength of the sphere, also the sign, strength and axis of the cylinder.

After both eyes have been examined the spherocylindrical combinations should be placed in the trial frame and the patient examined at the test type as frequently the plus may be increased and the minus decreased.

310. Stenopaic Slit Test for Astigmatism.—The stenopaic slit, Fig. 64, is an opaque disc with a narrow slit opening 1 or 2 mm. in width. It is frequently useful in making the examination for astigmatism.

Proceed as follows: Cover the eye not being examined with an opaque disc. In the outer cell of the trial frame place a plus lens of sufficient strength to fog the test type. In the inner cell place the slit. Rotate the slit through the various meridians until the patient states that he can read the type with greatest ease. You have now located the principal meridian of least error. Then place before the eye another plus sphere but of reduced strength. The first plus sphere should then be removed. Continue this process until best vision is obtained. Make a cross and on one arm write the sign and strength of the sphere, also the degree at which the slit stands.

Rotate the slit at right angles and correct this meridian in a similar manner. Write on the remaining arm of the cross the sign and strength of the sphere, also the degree at which the slit stands.

In the prescription state the sign and strength of each meridian, also the degree of each meridian.

311. Test for Presbyopia.—First test the vision for distance as in simple hypermetropia and simple myopia and astigmatism: The lens which the patient selects will be the distant correction. Record the findings. Then place in the patient's hands the reading chart in the correct reading position at a distance of thirteen or fourteen inches. Some patients, such as seamstresses, should be fitted at their favorite working distance.

The patient should be instructed to read the finest type possible while lenses of increasing strength are placed before the eye under examination. When fitted at the proper distance the type can be moved an equal distance on either side of this point before blurring takes place. This lens will be the nearby correction. Record the findings.

In hypermetropia a stronger lens is required for reading than for distance. In myopia a weaker lens is required for reading than for distance.

Bi-focals should be prescribed. The upper part for distance and the lower part for nearby work. Some people will, however, insist upon two pairs.

312. The Ophthalmometer.—The ophthalmometer is an instrument designed for the purpose of estimating the curvature of the cornea. If astigmatism exists the two principal meridians and axes may be quickly ascertained.

The cornea acts as a convex mirror and forms small erect images of distant objects. If the curve of one corneal meridian differs from the one at right angles it follows that they will form images of different size. The ophthalmometer is equipped with mires or targets that serve as the object. These images are then observed through a telescope which contains a double vision prism.

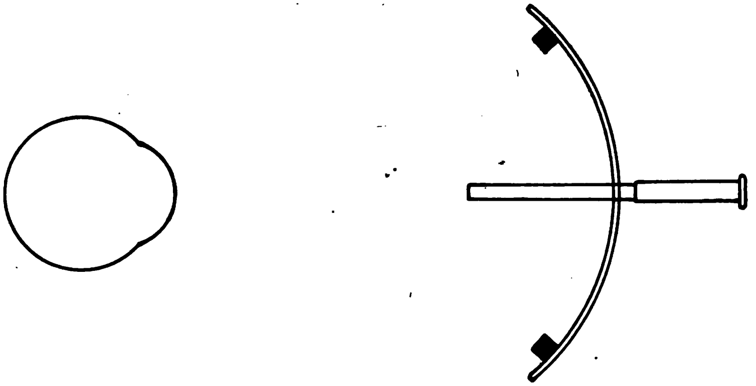


FIG. 170.

When the instrument is adjusted for a normal cornea, four images of the mires are observable; the two inner images are in contact with each other. Should the curves be abnormal there will be an overlapping of the images. The dioptric value of these curves is registered by the instrument.

The radius of curvature of the cornea may be derived from the formula, Fig. 170:

$$r = \frac{2ab}{o-2b}$$

Let *a* = the distance of object from cornea, *b* = size of corneal image. o = size of object. Size of object = distance between *ab*. Size of image = distance between *ab*. *ab*. is estimated by the telescope and recorded. Figs. 171, 172, are a front and back view of the *C*-1 ophthalmometer which is a very highly developed instrument. Briefly the manfuacturers make the following claims: A one position ophthalmometer, *i.e.*, two separate pairs of mires—a separate pair for each principal meridian. Once the primary position is located no further change in the position of the mires or

FIG. 171.

the telescope is required. The possibility of the movement of the eye under examination when the mires are moved to the secondary position is therefore entirely eliminated. The principal meridians are located by rotating the mires through an arc of only 90.° The astigmatism is quickly and accurately measured.

FIG. 172.

Irregularities of the corneal surface are easily diagnosed. Renders the mydriatic unnecessary in most cases. Achromatic objectives and prisms, etc. I am indebted to The F. A. Hardy Co. for the following plates.

Fig. 173 represents the images of the mires as seen reflected from a cornea when the eye piece is in the primary position. They are shown as widely separated, but are in perfect alignment as when reflected from a perfectly spherical cornea or an astigmatic cornea at an exact axis.

Fig. 173.

Fig. 174.

Fig. 174 represents the images of the mires as seen reflected from an astigmatic cornea when they lie midway between its two principal refracting meridians. The toric surface of an astigmatic cornea displaces one mire above and the other below the central plane of the two, except when they lie exactly in the plane of one of its principal meridians.

Fig. 175.

Fig. 176.

Fig. 175 represents the images of the mires as seen reflected from a cornea when the eye piece is in the primary position after the adjustments for both axis and radius have been made or after a perfect cross has been formed. The reading in the primary position is made at this point. The solid meridian pointer indicates the meridian that has been measured.

Fig. 176 represents the images of the mires as seen reflected from an astigmatic cornea, after the eye piece has been rotated to the secondary position, but before the adjustment to form a cross has been made.

After the latter adjustment has been made the readings in the secondary position may be noted. These readings are the radius of curvature in the secondary position. The dioptric value of that radius, the location of both principal meridians, the exact amount of corneal astigmatism and whether it be with or against the rule.

The ophthalmometer has been attacked on the grounds that it measures the corneal curves only and that the lenticular astigmatism may compensate for the unequal corneal meridians. In view of this argument the ophthalmometer findings cannot be taken as conclusive. Usually, however, the two principal meridians can be quickly found and approximately measured, even if lenticular astigmatism should exist. The refractionist is thus enabled to place before the eye a close approximation of the full correction without waste of time and needless tax on his patient.

The popularity of the ophthalmometer is demonstrated by its wide use.

CHAPTER IX

RETINOSCOPY

313. **Retinoscopy,** Sometimes termed "Skiascopy," "Keratoscopy," "Shadow Test," etc. An objective method of fitting eye glasses.

The operator selects the correcting lens, unaided by the patient's statements as to how well he sees.

314. **Retinoscope.**—Plane and concave.

FIG. 177.

The retinoscope is an instrument employed to ascertain the refractive condition of the eye. It consists of a small, round mirror, with a small transparent opening in the center, which opening is referred to as the "peep hole," "sight hole," "optic aperture," etc. This opening is sometimes drilled through the glass, but it is preferable to simply have the quicksilver removed, as additional reflecting surface is obtained, and an image of the opening eliminated. The opening ranges from 2 to 5 mm. diameter. mm. = 1/25 inch approx. See Sec. 377. The mirror should be of good quality, and fixed, to a black surface. Fig. 177. To the mirror is attached a handle. It is claimed that more precise results are obtained with the plane mirror as the shadow is better defined.

The axis is an imaginary line extending through the mirror and handle. The retinoscope is held by the handle and when turned or twisted by the fingers the mirror is said to be "tilted," "revolved" or "rotated" on its axis. The mirror tilts in the meridian at right angles to the meridian in which the axis stands, *e.g.*, if the refractionist holds the mirror before his eye, with the handle perpendicular (90°) and turns the handle the mirror will be tilted in the horizontal meridian (180°). He would therefore be examining the 180th meridian of the patient's eye.

315. The Necessary Equipment.—To practice retinoscopy in addition to the retinoscope it is necessary to have a darkened room, a lamp and trial frame. An adjustable chair is important.

FIG. 178.

316. Geneva Combined Retinoscope and Ophthalmoscope. The manufacturers make the following claims for this instrument. Fig. 178. A large clear view of the retina is obtained. Can be instantly converted into a retinoscope and the optical condition of the eye quickly determined whether emmetropic, hypermetropic, myopic, or astigmatic, including the axis. It is easily operated. No dark room required. It is operated on city current or by batteries. A very fine instrument.

317. Self-luminous De Zeng.—The De Zeng Self-luminous Retinoscope, Fig. 179, is an instrument of merit. It has an arrangement whereby the volume of light may be controlled at will. It operates on dry batteries contained in the handle. Ordinary lamp troubles are eliminated. The De Zeng is a very popular instrument as it should be.

FIG. 179.

318. The Dark Room.—The darker the room, the better defined will be the fundus reflex. Total darkness, however, is unnecessary.

Under ordinary circumstances if the shades are closely drawn, the room will be sufficiently dark. Pains should be taken that it is well ventilated, and the temperature of a comfortable degree.

319. The Lamp.—The light, Fig. 180, should be clear, steady and white. It should be of such intensity that when reflected into the patient's eye, it will illuminate the retina sufficiently so that the reflected rays will form a bright image upon the observer's retina. It should not be so bright as to cause discomfort.

The light should be screened, so that it can emerge from the jacket at but one point, opposite the broad side of the flame. The size of the opening is to be regulated by a diaphragm. The diaphragm in Fig. 180 is excellent. Some lamps are provided with an Iris diaphragm. The opening should be adjustable from about 5 to 25 mm. One mm. = 1/25″ approx. in diameter. In making an examination, the different sized apertures should be given trial, as under different conditions, all sizes have their advantages. As a rule, when near the point of reversal the smaller apertures will afford best results.

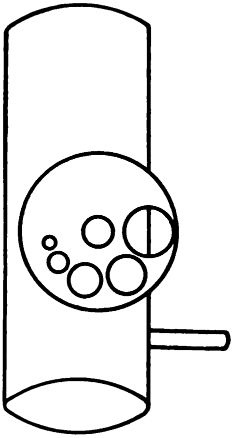


FIG. 180.

To prevent excessive radiation of heat waves, the jacket, if a metal one, should be provided with an asbestos jacket.

Oil, gas and electricity are used. If kerosene, a tablespoon, of common salt will whiten the flame considerably. The Argand and Welsbach burners are the favorites of many refractionists.

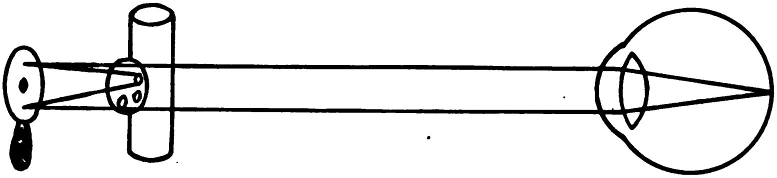


FIG. 181.

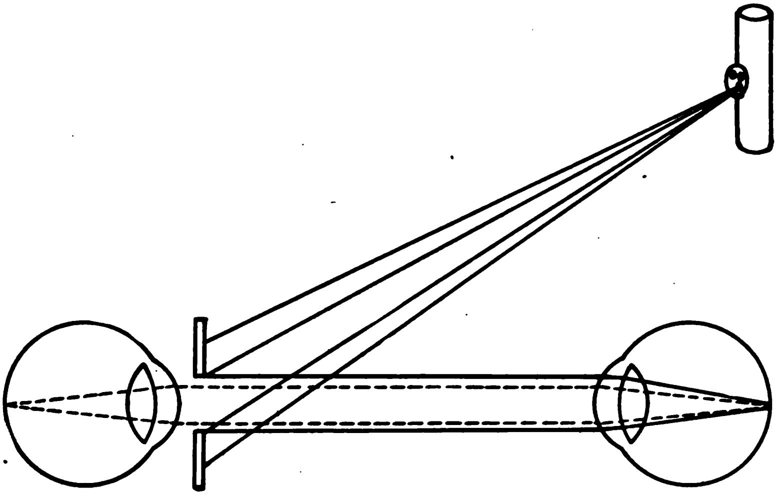


FIG. 182.

The lamp should be supported by a pedestal, or fastened to the wall by an adjustable bracket.

The lamp can be placed as follows: About six inches in front of and a little to one side of the refractionist's head. Fig. 181. If the observer uses his right eye the lamp should be on his left side. If the mirror is held before his left eye, the lamp should be on his right side.

The flame peep hole and patient's eye should be on a level. Unless the distance is very short, there may appear on the patient's pupil a small black dot, which is an image of the peep hole.

Or the lamp can be placed above the patient's head. Fig. 182. If the concave mirror is used it should be borne in mind that with

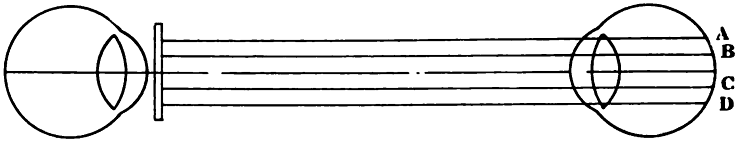


Fig. 183.

the light at a distance of one meter, 39.37 inches, that the light from it, which is reflected into the patient's eye, will be comparatively feeble. Also that the patient may suffer discomfort from the heat if excessive.

320. Trial Case and Trial Frame.—The ordinary trial case and trial frame will answer the purpose.

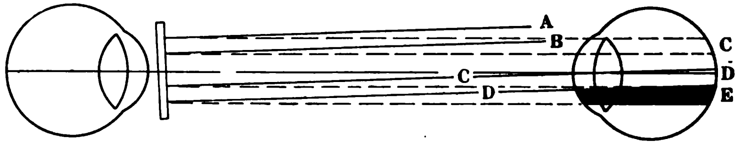


Fig. 184.

321. Adjustable Chair.—As the patient's eye should be on a level with that of the refractionists, a chair adjustable for height should be a part of the equipment.

322. How to Hold the Retinoscope.—The retinoscope should be held before the favorite eye, by the fingers, so that the sight hole will be opposite the pupil. It can be steadied by pressing the fingers against the face.

323. Basic Principles of Retinoscopy.—Retinoscopy is based upon the principle that if light is reflected into an eye (the patient's), Fig. 183, and an observing eye (the refractionist's), is placed in the direct path of the rays returning from it, these rays will form an image of the patient's retina upon the observer's. Then if the mirror is tilted, there will be a movement of this image noticeable to the observer, Figs. 183, 184, the appearance, direction and rate of movement depending upon the patient's ametropia.

Retinoscopy consists of finding the far point of an eye; a lens the focal length of which corresponds to the punctum remotum (far point of distinct vision), a lens that corrects the vision for distance.

324. Advantage of Retinoscopy.—Retinoscopy is a quick, accurate method of estimating an error of refraction.

In the case of children, the illiterate and malingerers its value can hardly be exaggerated, as the patient is asked no questions as to how well he sees.

325. Practice with the Retinoscope.—To obtain satisfactory results, the refractionist's vision should equal 20/20. If ametropic, his distant correction should be worn. The patient's accommodation should be suspended, and the pupil dilated. Fortunately the pupil usually dilates in a darkened room.

If the working distance is say twenty feet, it could be varied slightly without being taken into consideration. But if short, say one meter, pains should be taken that the distance is kept exact.

Why when the mirror is tilted, there is a movement of the "fundus reflex," the rate and direction depending upon the nature of the ametropia, embraces a knowledge of optics.

Figs. 183, 184, convey an idea as to why a shadow follows the illumination. When the mirror is held perpendicular with the retina, Fig. 183, the rays *A*, *B*, C, *D* enter and illuminate that part of the retina *A*, *B*, *C*, *D*. If the mirror is tilted, Fig. 184, the rays *A* and *B* do not enter the pupil. The rays *C*, *D* enter and illuminate but the portion *C*, *D*, leaving *E* in darkness. Thus the area *E* appears as a shadow. See Figs. 184, 186.

If the mirror was tilted sufficiently the rays *C*, *D* would not enter the eye, and the entire pupillary area would appear black.

If the working distance is 20 ft. or greater, the lens that neutralizes the retinal movement is the patient's correction for distance. But if the working distance is less, say 40 in., the far point is brought to 40 in. by the neutralizing lens. It must be removed to infinity. Therefore is required a +3.00 D. lens at 13 in. A +2.00 D. at 20 in., a +1.00 D. at 40 in., a +.50 D. at 80 in.

The foregoing table is of aid in writing the prescription. Thus *if the working distance* (distance between refractionist's and patient's eye) *is* 40 *inches and the neutralizing lens is plus, subtract* 1.00 D. *from it. If the neutralizing lens is minus, add minus* 1.00 D. *to it.*

If the neutralizing lens was say + 3.00 D. + 3.00 D. (−) 1.00 D. = + 2.00 D, hypermetropic, the patient's distant correction. If the neutralizing lens was − 3.00 D. adding − 1.00 D. to it − 3.00 D. (+) −1.00 D. = −4.00 D., myopic patient's correction for distance. If the neutralizing lens is +1.00 D. the eye is emmetropic.

If the neutralizing lens was plus, and of less strength than the +1.00 D, it would be necessary to subtract a larger from a smaller lens of like sign. Following are examples:

Neutralizing lens, +.75 D., correction −.25 D. Neutralizing lens +.50 D. correction −.50 D. Neutralizing lens +.25 D. correction −.75 D. A very simple rule is to always add −1.00 D. to the correcting lens. See "Transposition," Chapter XI.

No refractionist should undertake actual retinoscopy until after he has become thoroughly proficient through practice with the schematic eye.

Following is a description:

Practice With the Schematic Eye Plane Mirror.—The schematic or artificial eye is an ingenious instrument, Fig. 185. It should be in the possession of all studying retinoscopy. It consists of two telescoping tubes, at the end of one of the tubes is fixed a convex lens. This lens corresponds with the refractive media of the human eye. Also cells for the reception of trial lenses. This tube is also fixed to a base. At the back end of the other tube is a colored retina. This tube is graduated. When the tubes are telescoped to the zero mark the eye is emmetropic. If the tubes are telescoped, decreasing the distance between the lens and retina, the eye is made hypermetropic, the amount in diopters is indicated by a graduation at the end of the larger tube. If the tube containing the retina is drawn out beyond the emmetropic mark the eye becomes myopic.

Good practice is to set the eye at emmetropia. Place it at a distance of 20 feet, and reflect light into it. Rays returning from the colored retina and passing through the small round lens, will in turn pass through the peep hole and impinge upon the observer's retina. He will see a "reddish ball of light," designated as the "fundus reflex," "retinal illumination," etc., the appearance of which is similar to that of a cat's eye in the dark.

FIG. 185.

With the retina illuminated, next tilt the mirror. The ball of light will flicker and disappear in seemingly no direction and be replaced by darkness.

Most beginners will find difficulty in holding the retinoscope steadily enough to keep the retina illuminated.

Advisable practice is to reflect light on different objects in the dark room. A card measuring about one square foot should be fixed behind the eye. Reflect light on one side of the card and sweep it over the eye. This should be done very slowly.

To even get the retina illuminated and the eye in the path of the rays returning from it, i.e., see the "ball of light," is difficult for many.

A good plan is to hold the retinoscope so that the flame can be seen through the peep hole. Then if the retinoscope is being held before, say the right eye, with the handle perpendicular, close the left eye and turn the handle until the reflected light is swept over the eye to be examined. Open the left eye.

Next set the eye at hypermetropia 2.00 D. and when the fundus reflex is observed, tilt the mirror downward very slowly and the ball of light will move downward. Hence the expression "*with the mirror.*" This direction of movement is indicative of hypermetropia. The ball of light is followed by darkness called the "*shadow,*" Fig. 186, which spreads over the pupil as the mirror is being tilted. If the mirror is tilted sufficiently the entire pupil will appear black, because no rays enter to illuminate the retina.

Many beginners are unable to determine in which direction the ball of light moves, whether it moves in the direction the mirror is being tilted, or in an opposite direction.

The mirror should be tilted very slowly and the beginner should observe very carefully, not only the direction of movement, but also the rate of movement, having in mind the nearer emmetropic, the faster the "ball of light" travels.

A very helpful aid is to compare the movement of the "ball of light" with the movement of the light which is reflected upon the card, back of the schematic eye, around the pupil.

If the mirror is plane the light about the eye moves with the mirror. If the mirror is concave the light moves in a direction opposite to that of the mirror.

In actual practice the movement of the "ball of light" is to be compared with the movement upon the face about the eye.

The condition having been diagnosed as hypermetropic place in the cell a + 1.00 D. spherical lens. Then tilt the mirror downward. The "ball of light" still moves with the mirror only more rapidly. This increasing rapidity indicates that a partial correction has been made.

Replace the + 1.00 D. lens with a + 1.50 D. lens and tilt the mirror downward. The "ball of light" still moves with the mirror. Also with increasing rapidity.

Replace the + 1.50 D. with a + 1.75 D. lens and tilt the mirror downward. The "ball of light" still moves with the mirror, but it is followed so quickly by the shadow, that the effect is that of a flash and unless the mirror is tilted very slowly the beginner may not be able to observe the direction.

Next replace the +1.75 D. lens with a +2.00 D. *This is the neutralizing lens, the lens that corrects the eye for distance.* As before, tilt the mirror downward very slowly. The "ball of light" will flicker for an instant, and then disappear, seemingly in no direction. The *point of reversal* has been arrived at. If the +2.00 D. lens was replaced by say a +2.25 D. lens the eye would be over corrected; in other words, rendered myopic. The point of reversal would have been passed. When the mirror was tilted downward the "ball of light" would move upward. Hence the expression, *against the mirror*."

Then make the eye myopic say 2.00 D. The process of arriving at the neutralizing lens would be identical with that of finding the neutralizing lens in hypermetropia, excepting that minus lenses would be used.

Thus far, the working distance has been 20 feet. Because of this the lens that neutralized the retinal movement also corrected the ametropia for distance, and no deductions were necessary. But if the working distance was reduced to say 40 in. it would be necessary to subtract 1.00 D. from the neutralizing lens if it were plus, and add −1.00 D. if minus, as previously explained. Easier still, always add −1.00 D. to the neutralizing lens.

The working distance should be varied from 20 feet to a few inches, as this is valuable practice.

One meter is a favorite distance with many, as it is unnecessary to get up to change the trial lenses.

The eye can also be made to represent the various forms of astigmatism. A simple convex cylinder placed before the schematic eye produces simple hypermetropic astigmatism. A simple concave cylinder, simple myopic astigmatism. A plus cylinder and a plus sphere compound hypermetropic astigmatism. A minus cylinder and a minus sphere, compound myopic astigmatism. A convex sphere and a stronger minus cylinder or a minus sphere and a stronger convex cylinder, mixed astigmatism.

From the foregoing it can be seen that the schematic eye affords a study of the principles of retinoscopy otherwise unobtainable.

Retinoscopy With the Plane Mirror at One Meter

326. Arrangement.—Seat the patient comfortably, adjusting his chair so that his eye will be on a level with that of the refractionists. The refractionist should seat himself so that his eye will be one meter from and on a level with the patient's eye. The distance could be marked upon the wall or both hold the end of a meter stick. Fig. 181.

If the mirror is held before the right eye fix the lamp about six inches to the left and a little forward. Also the opening should be on a level with both eyes.

The trial case should be placed upon a small stand so as to be within convenient reach.

Next adjust the trial frame upon the patient's face. Before the eye to be examined, place a +1.00 D. spherical lens. It is customary to exclude the other eye with an opaque disc.

If atropinized the patient should look at the refractionist's forehead or preferably at the black edge of the retinoscope. The macular region will then be examined. During the test the patient should be instructed repeatedly to look in the direction designated. This is done to prevent accommodation as much as possible. The patient should not look directly at the mirror.

327. Fundus Reflex. Its Appearance and Movement In Emmetropia, Hypermetropia, Myopia and Astigmatism.—The necessary arrangement having been obtained reflect light into the patient's eye so that the fundus reflex can be viewed through the peep hole. The following information concerning the appearance of the "fundus reflex" will be helpful in making a diagnosis of the optical condition.

In emmetropia the color is of a reddish orange. The color decreases in brilliancy with increasing ametropia. In a high degree of ametropia it is greyish. The fundus reflex is brighter in albinos, because in brunettes there is more absorbing pigment.

In emmetropia, simple hypermetropia and myopia, the entire pupillary area is of a uniform color and shade, Fig. 186.

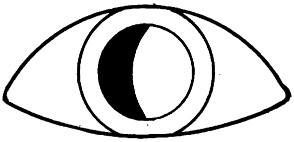


FIG. 186.

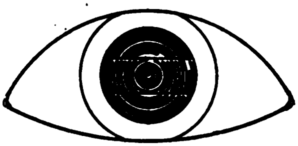


FIG. 187.

In regular astigmatism there may extend across the pupil a "band" of a lighter or darker shade, Figs. 187, and 188.

A peripherial ring is evidence of spheric aberration.

Two concentric rings indicates conic cornea.

Irregular lenticular astigmatism is indicated by striations radiating from the center toward the periphery of the pupil.

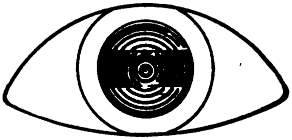


FIG. 188.

328. The Examination.—With the foregoing borne in mind, tilt the mirror in the different meridians. If the fundus reflex simply flickers and disappears in no traceable direction the eye is emmetropic. But if on tilting the mirror the fundus reflex moves with the mirror in all meridians, and the rate of movement is the same, it is a case of simple hypermetropia. If the movement is opposite the movement of the mirror in all meridians and the rate of movement the same the condition is simple myopia. If a band of light appears as in Figs. 187, 188 or there is simply a difference in the rate of movement or direction in the two principal meridians, the eye is astigmatic. Thus from the foregoing:

If the working distance is one meter and the patient's accommodation is suspended, the optical condition of his eye can be quickly ascertained, whether emmetropic, hypermetropic, myopic, or astigmatic.

Following is the procedure of examining a hypermetropic, myopic or astigmatic eye:

329. Test for Simple Hypermetropia.—Place a + 1.00 D. lens before the eye to be examined. If the fundus reflex moves with the mirror, replace the + 1.00 D. with say a + 1.50 D. If this lens does not neutralize the fundus reflex replace it with another stronger one. Continue this process of placing before the eye plus spherical lenses of increasing strength until the point of reversal is arrived at.

At this period two things should be carefully ascertained. First, that the retinal movement is accurately neutralized, and second, that no astigmatism, however slight, is present.

Therefore, replace what is adjudged to be the neutralizing lens with one say + .50 D. stronger. This lens should produce a reversal of the movement. In order to select the correct lens the mirror should be tilted very slowly and the refractionist with both eyes open observe carefully.

Unless the mirror is tilted very slowly in all meridians a slight amount of astigmatism may not be detected.

If a plus spherical lens + 1.00 D. neutralizes the movement, the eye is emmetropic. If the neutralizing lens is stronger than + 1.00 D. the eye is hypermetropic.

330. Test for Simple Myopia.—If the "ball of light" moves against the mirror, replace the + 1.00 D. with a + .75 D. or a + .50 D. or a + 25 D. spherical. If none of these neutralize the movement, commence with a weak minus spherical, replacing it with stronger ones until the correct lens is found.

331. Test for Regular Astigmatism.—If with the + 1.00 D., before the eye, a band appears, the band of light is in line with the meridian of least ametropia; hold the axis of the mirror so that the two will be in line. Notice on the trial frame the degree. Then make a cross. One line represents the degree of the band, also one of the two principal meridians.

Next make another line at right angles to the first. This line represents the other principal meridian.

Or it may be that no band will appear, but that as the mirror is tilted in the different meridians, there will be noted a difference in direction.

A simple plan is to ascertain the degree of one of the two principal meridians. If a band is present hold the handle of the mirror in line with it. Tilt the mirror and observe the effect in the meridian at right angles, as this is the one being examined. If it is hypermetropic or myopic correct it, as in the test for simple hypermetropia and simple myopia. The light moved with the mirror in the 75th meridian. A +4.00 D. sphere neutralized it. On the line which represented the 75th meridian was written + 4.00 D. This lens was removed and replaced by a +1.00 D. Then the axis was placed at right angles to its former position, *i.e*, at 75°. The mirror was tilted and the movement in the 150th meridian was with the mirror. A +2.00 D. sphere neutralized it, so +2.00 D. was written on the line marked 150°.

To convert these findings into a spherocylindrical combination see "Transposition," Chapter XI.

If no "band" appears but a specific movement is observed in some meridian, hold the handle of the instrument either in line with it or at right angles. The two principal meridians having been located, as previously described.

The retinoscopic or dark room findings should be converted into a spherocylindrical combination, which combination should then be placed before the patient's eyes and the mirror tilted in all meridians. If all meridians are neutral the test may be considered a correct one.

The patient should then be allowed to read with the combinations for a half hour or so. See "Transposition," Chapter XI.

332. Retinoscopy with the Concave Mirror.—The concave mirror is not recommended because of the more inaccurate results. However, if used, the lamp should be over and at a distance of about one meter back of the patient. The diaphragm opening should be about 30 mm., Fig. 182.

The mirror must be of such focal length that the rays will cross before entering the patient's eye. Thus the "shadow" will move opposite to that produced by the plane mirror.

CHAPTER X

OPHTHALMOSCOPY

333. *The refractionist judges which lens corrects the patient's ametropia. In this selection he is guided by the course of the emergent rays. The patient is asked no questions as to how well he sees. Therefore, ophthalmoscopy like retinoscopy is an objective method of fitting glasses.*

Following is a description of the ophthalmoscope, Fig. 189.

Like the retinoscope, its construction is based upon the principle that if the retina (the patient's) is illuminated and an observing eye (the refractionist's) is placed in the direct path of the rays returning from it, an image of the patient's retina will be formed upon the observer's.

FIG. 189.

In mechanical construction it is similar to the retinoscope. It consists of a small mirror, plane or concave. In the center is a small peep hole. At the back of the mirror are arranged revolving discs. The periphery of these discs contain small plus and minus spherical lenses. These discs are so arranged that when revolved, the lenses can be placed over the peep opening. The lenses range from .50 D. to approx. 16 D. However a combination of 24 D. may be obtained.

The purpose of the ophthalmoscope is three fold, viz.: *To ascertain the refractive condition of the eye. Examine the refractive media for opacities. The retina for pathological conditions.*

334. Basic Principles of the Direct and Indirect Methods.—To comprehend the basic principles of these two methods, involves a knowledge of the formation of images by a convex lens. The refractive media of the eye is likened to a convex lens, and the retina to the object.

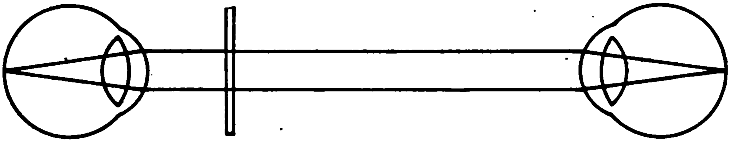


FIG. 190.

When the patient's retina is illuminated by light reflected from the ophthalmoscope, the rays which return from the illuminated retina, will emerge parallel if it is situated at the principal focus of the refractive media, as in emmetropia, Figs. 190, 193. If the retina is within the principal focus of the refractive media, as in hypermetropia, Figs. 191, 194, the rays emerge divergent. The image formed back of the retina is virtual and enlarged.

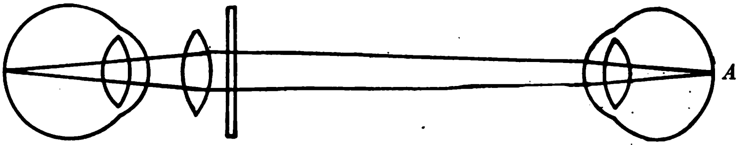


FIG. 191.

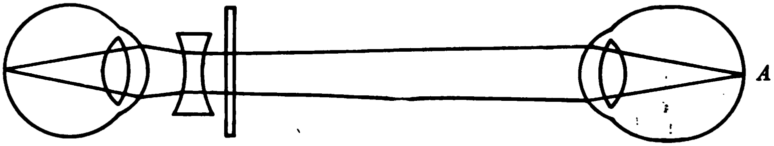


FIG. 192.

If the retina is outside the principal focus as in myopia, the rays emerge convergent, Figs. 192, 195, a real inverted image is formed in the air.

In the direct method, the lens in the ophthalmoscope which renders the emergent rays parallel is a measure of the ametropia. This lens also renders the details of the fundus clearest. It being a fact that unless the emergent rays enter the observer's eye parallel, the fundus will appear hazy.

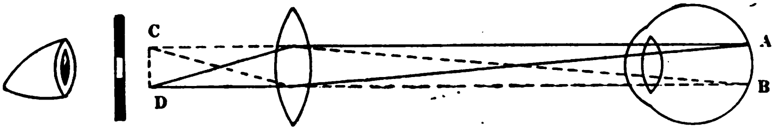


FIG. 193.

In emmetropia the rays emerge parallel. Consequently a good view of the fundus is obtained without using any of the lenses in the ophthalmoscope.

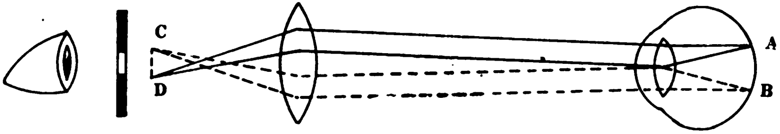


FIG. 194.

In hypermetropia, the rays emerge divergent. To render them parallel a plus lens is placed over the peep hole.

In myopia, the rays emerge convergent. To render them parallel a minus lens is required.

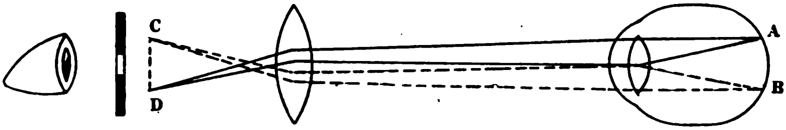


FIG. 195.

In the indirect method any increase or decrease which takes place in the size of the retinal image, which is formed by a convex lens, placed close to the eye under observation and then receded, is an indication of the kind and degree of the ametropia.

335. Dark Room, Light and Refractionists' Accommodation. As in retinoscopy a darkened room and screened light are quite essential. To obtain accurate results the accommodation of both refractionist and patient must be suspended.

Many refractionists can learn to relax their own at will by the following procedure:

Hold an object, such as a printed page, several inches away. Try to imagine that it is at infinity and that while looking at it you are simply staring into space. You will find that as the accommodation relaxes, the type becomes indistinct, the pages finally appearing of a grayish shade. Any accommodation renders the type more distinct. Such practice should be had until the refractionist has full control of his accommodation, before attempting actual ophthalmoscopy. Practice with the schematic eye is recommended.

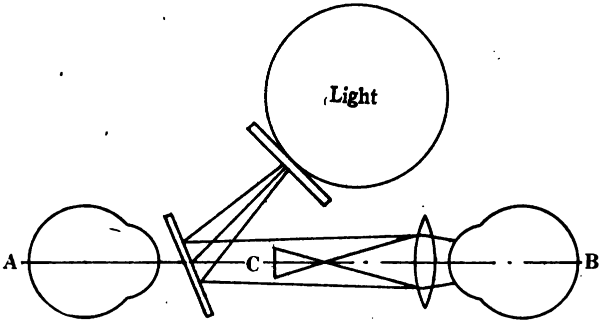


FIG. 196.

If the refractionist has an error of refraction it must be taken into consideration when writing the prescription, unless the full distant correction is worn.

336. The Indirect Method.—The light may be placed as in Fig. 181; the patient should look straight ahead into space and never at the mirror. Fig 196, is a plan (birds-eye) view of the arrangement. Light is reflected into the eye *B*. An image is formed in the air at *C*, which is observed by the eye at *A*.

The refractionist takes a position about twelve inches from the patient. With one hand he holds the ophthalmoscope before his own eye. A simple retinoscope will serve the purpose. With the other hand he holds a large convex lens of 13.00 D. or 20.00 D., at the focal distance of the lens from the eye to be examined. The lens can best be steadied by resting the fingers upon the patient's brow. Light is then reflected into the patient's eye until a clear view of the image is obtained. The image is largest in hypermetropia, and smallest in myopia. Then the convex lens is slowly receded. *If the size of the image remains unchanged, the eye is emmetropic, Fig.* 193. *If the image decreases, hypermetropic, Fig.* 194. *If it increases, myopic, Fig.* 195.

If one meridian remains unchanged in size, and magnification takes place in the meridian at right angles, it is a case of simple myopic astigmatism. If diminution only takes place in one meridian, it is simple hypermetropic astigmatism.

If unequal magnification takes place in the two principal meridians, the condition is one of compound myopic astigmatism. If unequal diminution, it is compound hypermetropic astigmatism.

If magnification takes place in one meridian, and diminution in the meridian at right angles, it is a condition of mixed astigmatism. The higher the degree of ametropia, the more rapid will the image change in size.

Astigmatism is evidenced by the optic disc appearing oval instead of round.

Troublesome images of the flame can be removed by tilting the lens slightly. The image of the peep hole can be eliminated by slightly rotating the mirror.

A large plus lens of 13.00 D. or 20.00 D. is placed close to the eye being examined, and light from the mirror reflected into it. If the eye is emmetropic, Fig. 194, the retina, *A*, *B* being at the focus of the refractive media, the rays from the eye will emerge parallel, and entering the plus lens parallel will be refracted as rays from infinity. Therefore a set of rays from *A*, *B* will be brought to a focus at the principal focus of the lens. At *C*, *D* an image of the retina is formed in the air, observable through the sight hole in the mirror.

If the retina is within the principal focus of the refractive media, which it is in a hypermetropic eye, Fig. 194, the rays emerge divergent. Because of this, the plus lens brings them to a focus outside of the principal focus as at *C*, *D*, where an inverted image of the retina *A*, *B* is formed in the air.

If the retina lies outside the principal focus of the refractive media, Fig. 195, such being its location in a myopic eye, rays from the retina *A*, *B* emerge from it convergent. Because of impinging upon the plus lens convergent, they will be brought to a focus at a shorter distance than in emmetropia or hypermetropia. At *C*, *D* an inverted image of the retina is formed in the air.

The aerial image is larger in hypermetropia than in myopia. Its size and location depends upon the degree of the ametropia.

The size of the inverted image is smaller than that observed in the direct method, which is upright.

337. The Direct Method Ophthalmoscopy.—The patient should be seated comfortably, the stooping posture avoided.

Place the lamp on a level with the patient's ear, a little back of, and a few inches to one side, so that the pencil of light will not illuminate the eye to be examined, Fig. 181. It should be on the same side of the head as the eye being examined.

Fig. 197, is a plan (birds-eye) view of the arrangement. Light is reflected into the eye *B*. *A*, observes a magnified image.

The patient should be instructed to look straight ahead into space and not accommodate.

The refractionist seats himself on the right side of the patient when examining the right eye, holding the ophthalmoscope before his own right eye. When examining the left eye, he seats himself on the left side of the patient holding the instrument before his own left eye. The refractionist's eye should be slightly above the patient's. The ophthalmoscope should be as close to the patient's eye as his spectacle glass sets.

With the foregoing arrangement obtained and the accommodation of both parties suspended, reflect light into the patient's eye as in retinoscopy. An upright magnified virtual image of the patient's retina should be observed through the peep hole. For a fixation point, select a retinal vessel or the optic disc. In the macular region the vessels are few in number. Gaze, as in looking into space at this particular point, as the plus lenses from weaker to stronger, are turned over the peep hole. If the details become plainer, it is a case of hypermetropia. If the plus lenses cause the fixation point to become less distinct and minus lenses improve it, it is a case of myopia. If both plus and minus lenses obscure the details of the fixation point, the eye is emmetropic.

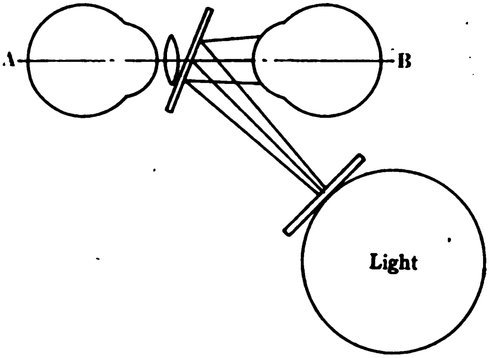


FIG. 197.

The lens in the ophthalmoscope which renders the details of the fundus clearest, is a measure of the patient's ametropia.

Estimating astigmatism with the ophthalmoscope is very difficult. A good plan to select for fixation points, are vessels in the horizontal and vertical meridians. If vessels in one meridian are plainer than those in another meridian, it is evidence of astigmatism. The lens which renders the vessels plain in one meridian, is a measure of the ametropia of the meridian at right angles. Thus, if say, with a plus 2.00 D. lens, the vessels in the vertical meridian are clear the horizontal meridian is hypermetropic 2.00 D

An oval instead of a circular disc is an indication of astigmatism. If ametropic the refractionist must subtract his own correction from the findings, unless his glasses are worn.

Light is reflected into the patient's eye, illuminating his retina. If his eye is emmetropic, Fig. 193, a set of rays from it say at *A*, will emerge parallel. Passing through the peep hole they will focus upon the observer's retina provided he is emmetropic. Because of this a clear view of the fundus results.

If the patient is hypermetropic, Fig. 194, rays from his retina will emerge divergent. From the diagram it can be seen that the set of rays *A*, *B* after emergence diverge. They would focus back of the observer's retina and form an imperfect image. Likewise the view of the fundus would be imperfect. A plus lens of proper strength over the peep hole causes the rays to become parallel, and a clear view of the fundus is obtained. This plus lens is a measure of the hypermetropia.

If the patient is myopic, Fig. 195, the rays emerge convergent, and would focus in front of the observer's retina. As in hypermetropia the view of the fundus is imperfect. If a minus lens of proper strength is placed over the peep hole they are caused to travel parallel and will then focus upon the observer's retina, affording a clear view of the fundus. This minus lens is a measure of the myopia.

CHAPTER XI

TRANSPOSITION

Prescriptions are transposed in order to obtain a periscopic lens and bring the cyl. axis at 90°. The axis when horizontal may interfere with convergence.

338. Positive and Negative Quantities. Transposition.—A convex lens converges parallel rays; magnifies objects; is referred to as a positive quantity, and is given the + sign. A concave lens diverges parallel rays; minifies objects; is referred to as a negative quantity and is given the − sign.

As the action on light of the two lenses is opposite it can be seen that if a plus lens was placed over a minus lens of equal strength, parallel rays would pass through the combination and emerge parallel. The minus lens would diverge the rays the same amount that the convex lens converged them. One lens overcomes, that is neutralizes the refractive power of the other. But if say a −2.00 D. lens was placed over a +5.00 D. lens, the −2.00 D. would neutralize 2.00 D. of the +5.00 D. leaving +3.00 D. unneutralized.

Therefore, adding quantities of unlike sign, subtracts from the refractive power.

If say a plus 1.00 D. lens was placed over a plus 2.00 D. lens or a minus 1.00 D. lens was placed over a minus 2.00 D. lens these combinations would have the same effect on parallel rays as a +3.00 lens or a −3.00 lens. Therefore adding quantities of like sign adds to the refractive power.

Algebraic addition and subtraction are based upon the foregoing principles.

339. Rule 1.—*Addition of Quantities of Like Sign.*—The values of positive or negative quantities is obtained by simple addition; give the result the sign common to both factors being added. Add as in simple arithmetic.

EXAMPLE:

+2.00 D.	−2.00 D.
+4.00 D.	−4.00 D.
+6.00 D. *Ans.*	−6.00 D. *Ans.*

340. Rule 2. *Addition of Unlike Quantities.*—Subtract the smaller number from the larger as in simple subtraction, give the remainder the sign of the larger number.

EXAMPLE:

+5.00 D.
−2.00 D.

+3.00 D. *Ans.*

341. Rule 3. *Subtraction of Quantities of Like and Unlike Sign.* Change the sign of the subtrahend and add it to the minuend.

Subtrahend is the number to be subtracted. Minuend is the number to be subtracted from. Remainder is the result.

EXAMPLE:

Minuend − 6.00 D
Subtrahend − 3.00 D.

Process:

−6.00 D.
−3.00 D.

−9.00 D. Ans.

EXAMPLE:

Minuend −5.00 D.
Subtrahend −3.00 D.

Process:

−5.00 D.
+3.00 D.

−2.00 D. Ans.

342. Transposition.—Transposition simply means to change the form of the original prescription, without changing the dioptric value of any meridian. The principal object of transposition is to secure a periscopic lens.

Little difficulty will be experienced in applying the following rules if the action of cylindrical and spherical lenses on light is thoroughly understood.

343. **Rule for Changing the Cylindrical Axis 90°.**—Do not alter the value of the cylinder, simply change its sign. Also change its axis, by 90°.

Always bear in mind that a cylinder has no strength in line with its axis, and that its greatest strength is at right angles to its axis. It then follows that if a meridian is to be changed by a cylinder, that the axis must be placed over the meridian at right angles.

Transposition of Sphero Cylindrical Combinations

344. **To Obtain the New Sphere.**—Add together the values of the old sphere and cylinder, applying Rules I and II. Take the result for the strength and sign of the new sphere for the transposed prescription.

345. **To Obtain the New Cylinder.**—Take the old cylinder simply change its sign and its axis by 90°.

EXAMPLE: +3.00 D. Sph. ○ +5.00 D. Cyl. Ax. 60°
Transposed +8.00 D. Sph. ○ −5.00 D. Cyl. Ax. 150°
EXAMPLE: +2.00 D. Sph. ○ −3.00 D. Cyl. Ax. 75°
Transposed −1.00 D. Sph. ○ +3.00 D. Cyl. Ax. 165°

346. **Converting Cross Cylinders into Sphero Cylinders.**

EXAMPLE:

−1.00 D., Cyl. Axis, 180° ○ +3.00 D., Cyl. Axis, 90°.

347. **To Obtain the New Sphere.**—Take one of the cylinders, say −1.00 D. Record it. This minus 1.00 D. sphere corrects the 90th meridian, and neutralizes 1.00 D. of the 180th. It can be plainly seen that the 180th meridian must now have +4.00 D.

348. **For the New Cylinder.**—Change the sign of the new sphere and then subtract it from the remaining cylinder according to Rule 3, thus:

+3.00 D. = +3.00 D.
−1.00 D. = +1.00 D.
―――――――
+4.00 D. Cylinder

The new cylinder is plus 4.00 D. Place the axis over the meridian fully corrected by the sphere which is 90°. Prescription reads − 1.00 D. Sph. ⊂ +4.00 D. Cyl. Axis, 90°.

Before transposing the following findings or prescriptions it is necessary to convert them into sphero cylindrical combinations. Subjective findings, retinoscope findings and torics, including simple cylinders and cross cylinders.

349. Rule for Converting Simple Cylinders into Sphero Cylindrical Combinations.

350. **For the new sph.** take the strength and sign of the old cylinder.

351. **For the new cylinder** take the strength but opposite sign of the old cylinder and change its axis by 90°.

EXAMPLE:	Rx. +2.00 D. Cyl. Axis, 180°
Transposed	Rx. +2.00 D. Sph. ⊂ −2.00 D., Cyl. Axis, 90°
EXAMPLE:	Rx. −4.00 Cyl. Axis × 90°
Transposed	Rx. −4.00 D. Sph. ⊂ +4.00 Cyl. Axis, 180°

352. **Torics, Finding the Strength of the Two Principal Meridians.**—By neutralization or with a lens measurer, find the strength of the two principal meridians. If a lens measurer has been used, subtract from the meridians on the toric surface the dioptric power of the other surface, applying Rule 2. Then record the strength of each meridian.

353. Reducing Toric Findings to a Sphero Cylindrical Combination.

EXAMPLE:

+2.00 D. 90th meridian ⊂ +3.00 D. 18th meridian.

These figures represent the actual strength of each meridian.

354. **To Obtain the New Sphere.**—Take say + 2.00 D. Record it.

355. **To Obtain the New Cylinder.**—Change the sign of the new sphere and add it to the meridian at right angles, applying Rule 2. −2.00 D. (+) + 3.00 D. = + 1.00 D. Cylinder. The 90th meridian was corrected by the sphere; therefore, place the axis at 90°.

Converted prescription now reads:

+2.00 D. Sph. ⊂ +1.00 D. Cyl. Ax. 90.°

356. Converting Retinoscopic Findings into Sphero Cylinder Combinations.

EXAMPLE:

+3.00 D. 90th meridian, − 3.00 D. 180th meridian.

357. To Obtain the New Sphere.—Take the strength of say the 90th meridian, which is +3.00 D. Record it.

358. To Obtain the New Cylinder.—Take the new sphere, change its sign which makes it −3.00 D., and add it to the meridian at right angles, applying Rule 2. Result = −6.00 D. Cylinder. The 90th meridian was corrected by the sphere, therefore place the axis at 90°.

Converted findings read:

+3.00 D. Sph. ⊂ 6.00 D., Cyl. Ax. 90.°

359. Rule Converting Prescriptions and Sphero Cylindrical Combinations into Torics.—First, determine the dioptric power of each meridian. Next add to both meridians a like amount of plus (6.00 D. is preferable as stock torics usually have a 6.00 D. base curve). To the inner surface add the same amount of minus.

EXAMPLE:

+3.00 D., 165th meridian −4.00 D.; 75th meridian.
+3.00 D. (+) (+6.00 D. = +9.00 D. and − 4.00 D (+) +6.00 D.
= +2.00 D.

For the inner surface specify a minus 6.00 D. sphere. It is plain that the two meridians remain unchanged in strength and the toric form is obtained. Better practice is to give the strength of the principal meridians and then specify toric in the prescription.

CHAPTER XII

FITTING OF FRAMES AND MOUNTINGS AND WRITING PRESCRIPTIONS

360. The spectacle frame should fit accurately and comfortably, or the full measure of benefit will not be derived from the lenses, even if they correct the error of refraction. From what we have learned of the prismatic effect of decentered lenses, it is self-evident that the optical center of the glass should center with the pupil. If the patient is astigmatic and demands a nose glass known as a finger piece mounting, he should be cautioned that it is impossible to keep the cylinder axis in a fixed position, and that much needless discomfort will result.

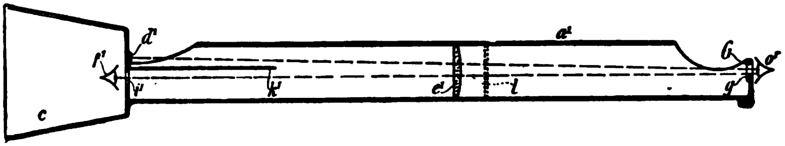


FIG. 198.

The Bausch & Lomb Interpupillary Gauge, Fig. 198, is a valuable addition to any refractionist's outfit.

361. **Interpupilliary Gauge.**—This instrument has been designed with a special view to the requirements of practitioners and provides a reliable, convenient, and time saving apparatus for measuring the distance between the pupils, when vision is directed upon distant or near objects.

The peculiar advantage of the instrument is that it measures the distance of either pupil from the center of the bridge of the nose, and thus takes into account the almost universally existing asymmetry of the eyes with respect to that point. In this way a most important measurement is obtained for the accurate adjustment of a spectacle frame.

The measurement involves no setting of a scale and in the majority of cases a single reading suffices to furnish the required information. Nevertheless we would advise one to take two or three readings, of which the average is to be taken for the measurement.

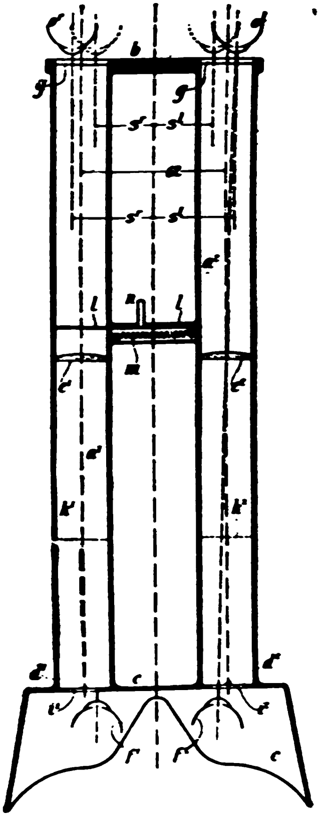


FIG. 199.

362. Working Principle of the Instrument.—The tubes a^1, and a^2, as shown in Figs. 216–217 contain two lenses, e^1, and e^2, and at their focal planes two identically similar fixation marks d^1 and d^2, of which they form virtual images at a distant point on the axis of the lenses. When the patient fixes the image of the mark d^2, which he can see with the left eye o^1, the axis of this eye will be parallel to the axis of the lenses e^2, *i.e.*, it looks straight ahead, and this would be the case whether the distance between the patient's pupil $s^r + s^1$, is equal to, greater, or less than the distance of the axis of the lenses.

The operator now pulls the shutter slide 1, by the pin *n*, against the tension of the spring *m*, toward the right until a^1 opens, whilst a^2 closes. The patient should then look with the right eye or at the distant image of the fixation mark d^1, and the latter will be seen under the same conditions as in the case of the left eye. The patient is by this means enabled to separately direct either eye straight forward, and the readings of the *position* of the pupils are taken one after the other. On either side of the bridge *b*, which is to be rested on the bridge of the nose of the patient, is a scale *g*, appearing below the center of the pupils and another scale *G*, appearing above the pupils, as shown in Fig. 200.

The scales are placed in such a position that the reading obtained represents the distance in millimeters from the center of the bridge *b*, to the center of each pupil separately. Scale *g* furnishes the distance from center of bridge to the pupils for reading glasses, and scale *G* for distance vision.

When the readings are taken the patient, is directed to look at the white mark seen in the instrument, and since in this manner he looks as though at an infinitely far object, the measurements obtained can be applied directly for the dimensions of the spectacle frame. The lower scale *g*, which furnishes the distance between the ophthalmic lenses, for reading, is of a special graduation, so that the direct reading from the scale represents the distance of each eye from the center of the bridge of the nose, for reading distance.

FIG. 200.

The magnifying effect of the lenses e^1 and e^2 renders it easier to take accurate readings. To insure that the eye of the patient may not be diverted from the image of the fixation mark, the operator's eyes f^1 and f^2, are shaded by a screen with two sight holes i^1 and i^2, which in turn are screened by two septa k^1 and k^2, at the back of the excisions through which light is admitted to the fixation marks, d^1 and d^2, within the tubes a^1 and a^2.

363. I must thank the F. A. Hardy Company for the following material.

1. The interpupillary distance (P. D.), that is, the space between the centers of the pupils (A. B.) Fig. 201.

2. The size and shape of the lenses. (Size of eye.)

3. The height of bridge measuring from the pupillary line (P. P. Fig. 201) to the lower edge of the crest of the bridge (C. Fig. 201).

4. The width of the bridge at its base, that is, the distance between the lower points of the bridge where they cease to touch the nose. (D. D. Fig. 201.)

5. The inclination meaning the position of the crest of the bridge, in relation of the central portion of the back plane of the lenses, when they are placed as near as possible to the eyes, without striking the lashes. This measurement is taken from the center of the back plane of lenses, (E. E.) Fig. 202 to upper edge of crest (L. Fig. 202).

6. The angle of crest, by which is meant the angle which the crest subtends to the plane of the temples, as shown in Fig. 202.

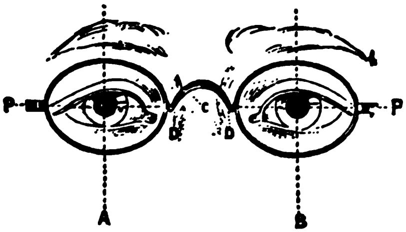


FIG. 201.

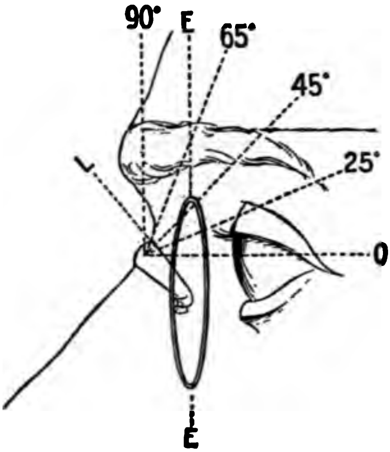


FIG. 202.

7. The distance between the temples measured at a point one inch back from the joints.

8. The temple length measured from the inner surface of the joint of the back of patient's ear. (See Fig. 203.)

9. The total length of temples.

10. The angle of joint, by which is meant the inclination of the joint to allow the lenses to be tilted for reading or other purposes.

In taking measurements for spectacle frames or mountings, a certain system should be followed, that is, the measurement should be taken up one after another in regular order. A set of trial frames, the dimensions of which are known, will greatly assist in getting the proper dimensions.

The first and most important of the above measurements is the pupillary distance (P. D.). This is best taken with a small vest pocket rule, measuring from the inner edge of the pupil of one eye to the outer edge of the pupil of the other.

If the glasses are for constant use, the patient should be looking at an object some distance beyond the operator, when the measurement is taken. The operator should hold the rule in his right hand, letting it rest upon the patient's nose, with its upper edge just below the pupils, and one end at the inner edge of the pupil of the right eye. Then note where the outer edge of the pupil of the left eye comes (or the edges of the iris may be used); by this method very exact measurements can be taken, especially if the operator will close his right eye when placing the rule and his left when noting the P. D. If the glasses are for reading, or close work only, the patient should be allowed to converge to the reading point when the measurement is taken.

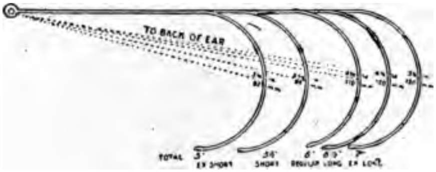


FIG. 203.

The second measurement to be considered is the size and shape of the lenses (known as size of eye). When considering this, it is well to remember that the total length of one lens and the distance between the two lenses should exactly equal the P. D. For instance, if the P. D. is 63 mm., and the distance between the eye wires of a spectacle frame, the bridge of which fits the patient correctly is 23 mm., the lenses must then be 40 mm. long. (For sizes of lenses see Sec. 142.)

The lenses should be as large as possible, or as large as compatible with the patient's features. This is especially true if peritoric, rimless or bi-focal lenses are to be prescribed. Rimless should be one size larger than rimmed lenses, and Toric should be one size larger than rimless or two sizes larger than rimmed. Half eye or crescent shaped lenses are desirable for presbyopes, especially so if no lens is required for distant vision. Short oval lenses should be prescribed if the nose is thick, or P. D. narrow. When a large lens is desired it may be provided for by bending the shanks of a saddle bridge in, as shown in Fig. 201, and for each millimeter they are bent in, the length of the lens should be increased two millimeters.

The third measurement to be noted is the height of the bridge. The correct height is best attained by the aid of the trial frames noted above. The center of the lenses should be slightly below the center of the pupils, when the patient is looking horizontally. A compromise between what might be considered the correct height for distance, and the correct height for near work, is what may be correct for glasses prescribed for constant use. Glasses used only for reading or close work should be about 2 mm. lower than those that are for constant use. Reading glasses should also be ordered slightly pantoscopic, but are not so made without special instructions.

The fourth dimension to be considered is the width of the bridge at its base. This measurement, like the third, is best obtained by the aid of trial frames with bridges of known widths. The contour of the nose and of the bridge should be the same. If it is the bridge will touch the nose at all points, thus distributing the weight of the spectacles over the nose more evenly than it would be if the base was too wide or too narrow. The lenses will also be held more rigid if the bridge fits correctly.

The fifth dimension to be noted is the position of the crest of the bridge in relation to the center of the back plane of the lenses (known as the inclination). Trial frames aid materially in getting this measurement. The lenses should be as near the eyes as possible, and not touch the lashes. If the lashes sweep the lenses when the bridge *rests at* the proper place on the patient's nose, the bridge shanks should be increased in length, just sufficient to have them clear. Peritoric lenses afford about 2 mm. more space for the eye lashes than ordinary lenses do.

The sixth dimension to be noted is the angle of the crest. As a rule an angle of 45° will rest comfortably on the nose, and if no angle is noted in the prescription such a one will be furnished. A bridge having such an angle will rest comfortably on a nose that does not vary more than 10° either way, from 45°, but an angle of 10° (nearly horizontal) or one of 80° (nearly vertical) may be required. Approximately the correct angle may be obtained by the aid of Dr. Young's crest cards, which may be had free upon request. The exact angle may be obtained with the Hardy crest and joint angle meter.

The seventh measurement to be noted is the distance between the temples at a point approximately one inch back of the joints, or where they would nearly touch the patient's face. The trial frames again aid us in getting the correct distance. Ordinarily the frames having the correct size of eye and length of bridge will be wide enough, and if they are, this measurement may be omitted; but with a narrow pupillary distance and wide face, the temples would bind. In such cases, they should be bent out, or, in extreme cases, an extra long end piece may be used.

The eighth dimension to note is the length of the temple to back of ear. The regular length of temple (6 inches), when curled, measures practically 4⅛ inches, from its joint end to the center of its curl. If this length is desired, this dimension may be omitted; but children often require only 3½ or possibly 3⅛ inches, while many adults require 4½, or possibly 5⅛ inches long to back of ear. This adjustment, like all others, should be correct. It can be made by curling the temples with the fingers.

The ninth dimension refers to the total length of the temples. If this length is given a temple can be supplied that can be quickly and easily adjusted as described above. Stock temples measure 5, 5½, 6, 6½, and 7 inches.

The tenth and last notation to be made refers to the joints. This is important only when it is desired to have the lenses tilted from the position they naturally take when the temples rest upon the patient's ears. As a rule the lenses are in the correct position when the temples are as above, but the ears may be too low. If so, the lenses will tilt out at the bottom, allowing the patient to look under them when looking down. This should be corrected by changing the angle of the joints to one that will bring the lenses to the correct position. If the glasses are to be worn for near work only, the joints should be slightly pantoscopic, or at an angle that will allow the patient to look straight through the lenses when reading—usually an angle of 10° will be sufficient.

364. Selection of Mounting and Lenses.—Not only should the frame fit accurately and comfortably, but it should be selected with the cosmetic effect in view.

Of all materials, gold is generally to be preferred on account of its strength, elasticity, non-tarnishing property and appearance. The aluminum frame is cheap and exceedingly light, therefore popular with a certain class. The silver frame is worn quite extensively. Steel frames are both cheap and strong, but rust easily. It is popular with some workmen.

In selecting the style of mounting the patient's age, occupation and social inclination should be taken into consideration. The size of eye should correspond with the patient's features, remembering that the larger size lenses in the periscopic or toric form should be prescribed when possible. Growing children should be given a rigid gold frame, and as a rule (particularly if the face is small) the size of the lenses should be smaller than those worn by adults. The rimless spectacle and nose glass are worn by young and middle aged people. The rimless nose glass is preferred by many young ladies. However, in selecting a rimless glass, it must be remembered that they are fragile as compared with the ordinary kind. Also the cylindrical axis is not held accurately in place. Rimless glasses are worn by *some* presbyopes. Possibly in many cases to conceal their age.

Whether the desired effect is obtained is a question. As a rule presbyopes select a neat substantial frame, with the lenses surrounded by rims.

As there is such an extensive variety of frames and lenses the refractionist will do well to study carefully the optical catalogues.

CHAPTER XIII

MATHEMATICS

365. The following list of mathematical signs and abbreviations are reproduced by permission from Machinery's Handbook, Copyright 1914, by The Industrial Press, New York.

366. Mathematical Signs and Commonly Used Abbreviations:

$+$	Plus (sign of addition)	g	Acceleration due to gravity (32.16 ft. per sec.)
$+$	Positive		
$-$	Minus (sign of subtraction)		
$-$	Negative	i	Imaginary quantity. $(\sqrt{-1})$
$\pm(\mp)$	Plus or minus (minus or plus)		
$\times$	Multiplied by (multiplication sign)	sin	Sine
		cos	Cosine
$\cdot$	Multiplied by (multiplication sign)	tan (tg) (tang)	Tangent
$\div$	Divided by (division sign)		
$:$	Divided by (division sign)	cot (ctg)	Cotangent
$:$	Is to (in proportion)		
$=$	Equals	sec	Secant
$::$	Equals (in proportion)	cosec	Cosecant
$\cong$	Approximately equals	versin	Versed sine
$>$	Greater than	covers	Coversed sine
$<$	Less than	$\mathrm{Sin}^{-1}a$	Arc the sine of which is a
$\geq$	Greater than or equal to		
$\leq$	Less than or equal to	arc sin a $(\mathrm{Sin}^{-1}a)$	Reciprocal of sin a $(1 \div \sin a)$
$\therefore$	Therefore		
$\sqrt{\ }$	Square root	sinh x	Hyperbolic sine of x
$\sqrt[3]{\ }$	Cube root	cosh x	Hyperbolic cosine of x
$\sqrt[4]{\ }$	4th root	$\int$	Integral (in calculus)
$\sqrt[n]{\ }$	nth root		
a^2	a squared (2d power of a)	$\int_b^a$	Integral between the limits a and b
a^3	a cubed (3d power of a)		
a^4	4th power of a	$!$	$5! = 1 \times 2 \times 3 \times 4 \times 5$
a^n	nth power of a	$\angle$	Angle
$\frac{1}{n}$	Reciprocal value of n	$\llcorner$	Right angle
		$\perp$	Perpendicular to
log	Logarithm	$^\circ$	Degree (circular arc or thermometer)
hyp. log	Hyperbolic, natural or Napierian logarithm		
nat. log		$'$	Minutes or feet
$\log_e$		$''$	Seconds or inches
ln		a'	a prime
lim.	Limit value (of an expression)	a''	a double prime
∞	Infinity	a_1	a sub one

α	Alpha	commonly used to denote angles	a_2	a sub two
β	Beta		a_n	a sub n
γ	Gamma		( )	Parentheses
θ	Theta		[ ]	Brackets
ϕ	Phi		{ }	Braces
Δ	Delta (difference)		B.H.P.	Brake horsepower
δ	Delta (differential)		B.T.U.	British thermal units
μ	Mu (coefficient of friction)		H.P.	Horsepower
π	Pi (3.1416)		I.H.P.	Indicated horsepower
Σ	Sigma (sign of summation)		K.W.	Kilowatt
ω	Omega (angles measured in radians)		M.E.P.	Mean effective pressure
			R.P.M.	Revolutions per minute
d	Differential (in calculus)			
e	Base of hyp. logarithms (2.71828)		C.G. S.	Centimeter-gram-second system

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367. These additional terms and explanations will be of value to many readers:

ab or $a.b.$ equals $a \times b$
1/Division sign. Example 8/4 equals 2
$\frac{a}{b}$ equals a/b equals a divided by b
$3 \div 4 = \frac{3}{4}$
0.5 equals five tenths
0.05 equals five hundredths
0.005 equals five thousandths
0.0005 equals five ten thousandths
0.00005 equals five hundred thousandths
0.000005 equals five millionths
5^2 equals 5 squared equals 5×5 equals 25
5^3 equals 5 cubed equals $5 \times 5 \times 5$ equals 125
′ ″ ‴ distinguishing marks as C', C'', C'''
Direct ratio of 25 to 5 is 25: 5 inverse ratio of 25 to 5 is 5 to 25

368. Solution of Right Triangles.—The solution of triangles is not a difficult thing. The table of Sines, Cosines, Tangents and Cotangents is reproduced by permission from Machinery's Reference Series No. 55, Copyright, 1912, by The Industrial Press, New York, N. Y. Those who wish to extend their knowledge should have a table of sines, tangents, etc.

Following is the solution of several problems in right triangles.

We know that a right triangle consists of 180 degrees. It then follows that one included angle, the right angle must be of 90 degrees. See Fig. 204. If we find Angle *A*, to equal 30 degrees the remaining angle must be 60 degrees.

369. Problem.—Find angle *A* (*C* divided by *D* equals sin of *A*) Fig. 204. Solution *C* divided by *D* equals 2.88675 divided by 5.7735 equals 5 answer. In the table under the column marked Nat. Sin. 5 stands opposite *O′* which indicates 30 degrees and no minutes. *A*, then is 30 degrees.

370. Problem.—Find angle *A* (*C* divided by *B* equals Tan. of *A*. Fig. 205. Solution: *C*, divided by *B*, equals 2.88675 divided by 5 equals .57735 *Answer*. In the column marked Nat. Tan. 57735 is opposite *O′*.*A*, is 30 degrees.

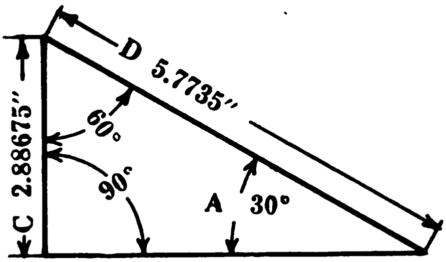

FIG. 204.

371. Problem.—Find angle *A* (*B* divided by *D* equals Cos. *A*). Fig. 206. Solution: 5 divided by 5.7735 equals .86603, *Answer*. See column marked Nat. Cos. .86603 is opposite *o′*. *A* is therefore 30 degrees.

372. Problem.—Find length of side *B* having *A*, and *D*. Fig. 207. Solution: (*D* × Cos. of *A* equals *B*) *D* equals 5.7735″ × the Cos. of 30 degrees which is .86603. The answer is 5″.

373. Problem.—Find length of side *B* having *A* and *C*. Fig. 208. Solution: (*C* × Cotangent of *A*) *C* equals 2.88675. Cotangent of 30 degrees equals 1.7320. The answer is 5″.

374. Problem.—Find length of side *C*. Fig. 209. Solution: (*D* × Sine of *A* equals *C*). Sine of 30 degrees equals .50000 *D* equals 5.7735, *Answer* 2.88675″.

375. Problem.—Find length of side *C*. Fig. 210. Solution: (*B* × Tangent of *A* equals *C*). Tangent of 30 degrees equals .57735. *B* equals 5. *Answer* 2.88675″.

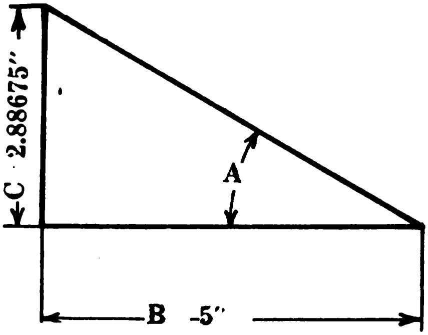

FIG. 205.

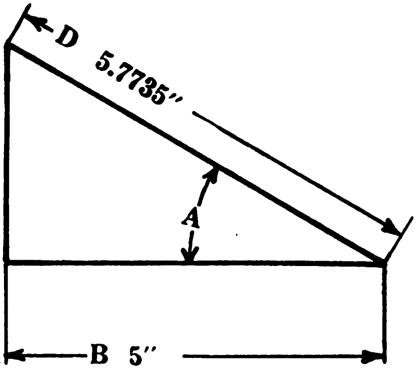

FIG. 206.

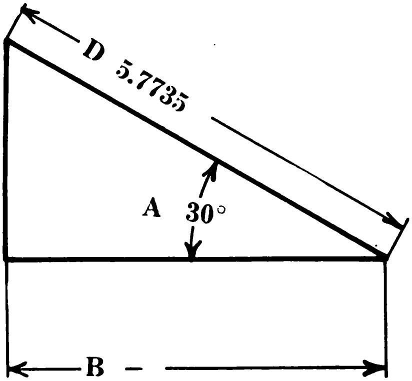

FIG. 207.

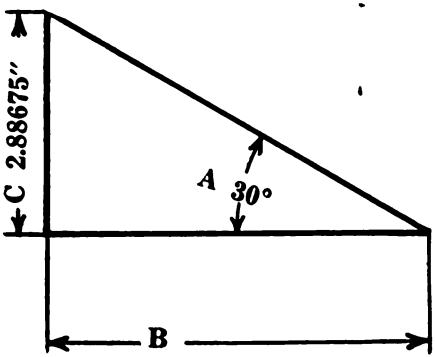

Fig. 208.

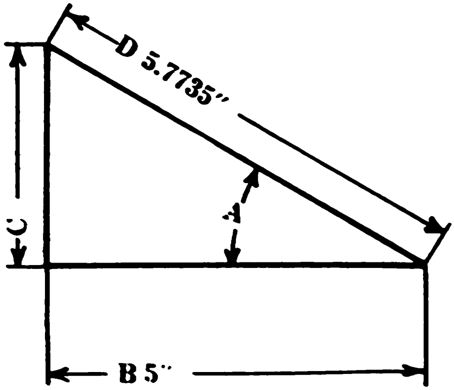

Fig. 209.

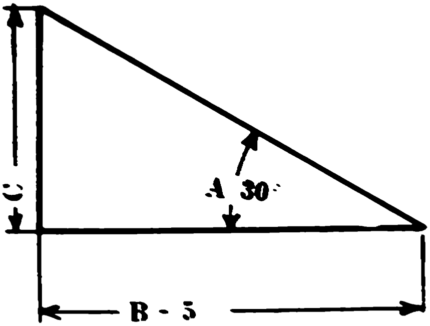

Fig. 210.

30°

′	Nat. Sin.	Log.	d.	Nat. Cos.	Log.	d.	Nat. Tan.	Log.	c.d.	Log. Cot.	Nat.	
0	50000	9.69897	22	86603	9.93753	7	57735	9.76144	29	0.23856	1.7321	60
1	025	9.69919	22	588	9.93746	8	774	9.76173	29	0.23827	309	59
2	050	9.69941	22	573	9.93738	7	813	9.76202	29	0.23798	297	58
3	076	9.69963	21	559	9.93731	7	851	9.76231	30	0.23769	286	57
4	101	9.69984	22	544	9.93724	7	890	9.76261	29	0.23739	274	56
5	50126	9.70006	22	86530	9.93717	8	57929	9.76290	29	0.23710	1.7262	55
6	151	9.70028	22	515	9.93709	7	968	9.76319	29	0.23681	251	54
7	176	9.70050	22	501	9.93702	7	58007	9.76348	29	0.23652	239	53
8	201	9.70072	21	486	9.93695	8	046	9.76377	29	0.23623	228	52
9	227	9.70093	22	471	9.93687	7	085	9.76406	29	0.23594	216	51
10	50252	9.70115	22	86457	9.93680	7	58124	9.76435	29	0.23565	1.7205	50
11	277	9.70137	22	442	9.93673	8	162	9.76464	29	0.23536	193	49
12	302	9.70159	21	427	9.93665	7	201	9.76493	29	0.23507	182	48
13	327	9.70180	22	413	9.93658	8	240	9.76522	29	0.23478	170	47
14	352	9.70202	22	398	9.93650	7	279	9.76551	29	0.23449	159	46
15	50377	9.70224	21	86384	9.93643	7	58318	9.76580	29	0.23420	1.7147	45
16	403	9.76245	22	369	9.93636	8	357	9.96609	30	0.23391	136	44
17	428	9.70267	21	354	9.93628	7	396	9.76639	29	0.23361	124	43
18	453	9.70288	22	340	9.93621	7	435	9.76668	29	0.23332	113	42
19	478	9.70310	22	325	9.93614	8	474	9.76697	28	0.23303	102	41
20	50503	9.70332	21	86310	9.93606	7	58513	9.76725	29	0.23275	1.7090	40
21	528	9.70353	22	295	9.93599	8	552	9.76754	29	0.23246	079	39
22	553	9.70375	21	281	9.93591	7	591	9.76783	29	0.23217	067	38
23	578	9.70396	22	266	9.93584	7	631	9.76812	29	0.23188	056	37
24	603	9.70418	21	251	9.93577	8	670	9.76841	29	0.23159	045	36
25	50628	9.70439	22	86237	9.93569	7	58709	9.76870	29	0.23130	1.7033	35
26	654	9.70461	21	222	9.93562	8	748	9.76899	29	0.23101	022	34
27	679	9.70482	22	207	9.93554	7	787	9.76928	29	0.23072	011	33
28	704	9.70504	21	192	9.93547	8	826	9.76957	29	0.23043	1.6999	32
29	729	9.70525	22	178	9.93539	7	865	9.76986	29	0.23014	988	31
30	50754	9.70547	21	86163	9.93532	7	58905	9.77015	29	0.22985	1.6977	30
31	779	9.70568	22	148	9.93525	8	944	9.77044	29	0.22956	965	29
32	804	9.70590	21	133	9.93517	7	983	9.77073	28	0.22927	954	28
33	829	9.70611	22	119	9.93510	8	57022	9.77101	29	0.22899	943	27
34	854	9.70633	21	104	9.93502	7	061	9.77130	29	0.22870	932	26
35	50879	9.70654	21	86089	9.93495	8	59101	9.77159	29	0.22841	1.6920	25
36	904	9.70675	22	074	9.93487	7	140	9.77188	29	0.22812	909	24
37	929	9.70697	21	059	9.93480	8	179	9.77217	29	0.22783	898	23
38	954	9.70718	21	045	9.93472	7	218	9.77246	28	0.22754	887	22
39	979	9.70739	22	030	9.93465	8	258	9.77274	29	0.22726	875	21
40	51004	9.70761	21	86015	9.93457	7	59297	9.77303	29	0.22697	1.6864	20
41	029	9.70782	21	000	9.93450	8	336	9.77332	29	0.22668	853	19
42	054	9.70803	21	85985	9.93442	7	376	9.77361	29	0.22639	842	18
43	079	9.70824	22	970	9.93435	8	415	9.77390	28	0.22610	831	17
44	104	9.70846	21	956	9.93427	7	454	9.77418	29	0.22582	820	16
45	51129	9.70867	21	85941	9.93420	8	59494	9.77447	29	0.22553	1.6808	15
46	154	9.70888	21	926	9.93412	7	533	9.77476	29	0.22524	797	14
47	179	9.70909	22	911	9.93405	8	573	9.77505	28	0.22495	786	13
48	204	9.70931	21	896	9.93397	7	612	9.77533	29	0.22467	775	12
49	229	9.70952	21	881	9.93390	8	651	9.77562	29	0.22438	764	11
50	51254	9.70973	21	85866	9.93382	7	59691	9.77591	28	0.22409	1.6753	10
51	279	9.70994	21	851	9.93375	8	730	9.77619	29	0.22381	742	9
52	304	9.71015	21	836	9.93367	7	770	9.77648	29	0.22352	731	8
53	329	9.71036	22	821	9.93360	8	809	9.77677	29	0.22323	720	7
54	354	9.71058	21	806	9.93352	8	849	9.77706	28	0.22294	709	6
55	51379	9.71079	21	85792	9.93344	7	59888	9.77734	29	0.22266	1.6698	5
56	404	9.71100	21	777	9.93337	8	928	9.77763	28	0.22237	687	4
57	429	9.71121	21	762	9.93329	7	967	9.77791	29	0.22209	676	3
58	454	9.71142	21	747	9.93322	8	60007	9.77820	29	0.22180	665	2
59	479	9.71163	21	732	9.93314	7	046	9.77849	28	0.22151	654	1
60	504	9.71184		717	9.93307		086	9.77877		0.22123	643	0
	Nat. Cos.	Log.	d.	Nat. Sin.	Log.	d.	Nat. Cot.	Log.	c.d.	Log. Tan.	Nat.	′

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376. Table of Decimal Equivalents of 8ths, 16ths, 32ds, and 64ths of an inch:

8ths

1/8 = 0.125
1/4 = 0.250
3/8 = 0.375
1/2 = 0.500
5/8 = 0.625
3/4 = 0.750
7/8 = 0.875

16ths

1/16 = 0.0625
3/16 = 0.1875
5/16 = 0.3125
7/16 = 0.4375
9/16 = 0.5625
11/16 = 0.6875
13/16 = 0.8125
15/16 = 0.9375

32ds

1/32 = 0.03125
3/32 = 0.9375
5/32 = 0.15625
7/32 = 0.21875
9/32 = 0.28125
11/32 = 0.34375
13/32 = 0.40625
15/32 = 0.46875
17/32 = 0.53125
19/32 = 0.59375
21/32 = 0.65625
23/32 = 0.71875
25/32 = 0.78125
27/32 = 0.84375
29/32 = 0.90625
31/32 = 0.96875

64ths

1/64 = 0.015625
3/64 = 0.046875
5/64 = 0.078125
7/64 = 0.109375
9/64 = 0.140625
11/64 = 0.171875
13/64 = 0.203125
15/64 = 0.234375
17/64 = 0.265625
19/64 = 0.296875
21/64 = 0.328125
23/64 = 0.359375
25/64 = 0.390625
27/64 = 0.421875
29/64 = 0.453125
31/64 = 0.484375
33/64 = 0.515625
35/64 = 0.546875
37/64 = 0.578125
39/64 = 0.609375
41/64 = 0.640625
43/64 = 0.671875
45/64 = 0.703125
47/64 = 0.734375
49/64 = 0.765625
51/64 = 0.796875
53/64 = 0.828125
55/64 = 0.859375
57/64 = 0.890625
59/64 = 0.921875
61/64 = 0.953125
63/64 = 0.984375

377. The Metric System of Measurement—Measures of Length:

1 Millimeter (mm.) = ... 0.03937079 inch, or about 1/25 inch
10 Millimeters = 1 Centimeter (cm.) = ... 0.3937079 inch
10 Centimeters = 1 Decimeter (dm.) = ... 3.937079 inch
10 Decimeters = 1 meter (m.) = ..39.37079 inches, 3.2808992 feet, or 1.09361 yards
10 Meters = 1 Decameter (Dm.) = ... 32.808992 feet
10 Decameters = 1 Hectometer (Hm.) = ... 19.927817 rods
10 Hectometers = 1 Kilometer (Km.)............ 1093.61 yards, or 0.6213824 mile
10 Kilometers = 1 Myriameter (Mm.) = ... 6.213824 miles
1 inch = 2.54 cm., 1 foot = 0.3048 m., 1 yard = 0.9144 m., 1 rod = 0.5029 Dm., 1 mile = 1.6093 Km.

378. Solution of Formulas in This Treatise.—Sec. 18. $V = \sqrt{1.41 \frac{P}{D}}$. Read: The velocity V is found as follows: P is divided by D, and then multiplied by 1.41. From this product we extract the square root and thus obtain V. Sec. 44. V equals $4dtr$. Read: V equals $4 \times d \times t \times r$

379. Extracting the Square Root.[1]—Assume that the square root of 119,716 is to be found. Beginning at the unit figure or decimal point, point off the number into periods of two figures each. Should there be an odd number of figures in the given number, the last period to the left will have only one figure.

Find the greatest whole number, the square of which does not exceed the value of the figures in the left-hand period (11), and write this number as the first figure in the root. In this example, this number is 3, the square of which is 9. Subtract this square from the left-hand period, and move down the next period of two figures and annex it to the remainder.

$$\begin{array}{rl} & 11'97'16 \;\underline{|346} \\ 3 \times 3 = & \underline{9} \\ 3 \times 20 = 60 & 297 \\ (60 + 4) \times 4 = & \underline{256} \\ 34 \times 20 = 680 & 4116 \\ (680 + 6 \times 6) = & \underline{4116} \end{array}$$

Multiply the figure of the root obtained by the constant 20 ($3 \times 20 = 60$); find how many times this product is contained in the number 297. This gives a trial figure for the second figure of the root; 60 is contained 4 whole times in 297, and 4 is, therefore, placed as the next figure of the root.

Subtract from 297 the product of 60 plus the figure of the root just obtained (4), multiplied by the same figure (4); $(60 + 4) \times 4 = 256$. If this product were larger than 297 it would indicate that the trial figure is too large, and a figure one unit smaller should be

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used. Then move down the next period of two figures and annex it to the remainder.

Multiply the figures of the root thus far obtained by 20; (34 × 20 = 680); find how many times this product is contained in 4116. This gives a trial figure for the third figure of the root. Then proceed as before.

If the last subtraction leaves no remainder, and if there are no more periods of figures to move down from the given number, the obtained root 346 is the exact square root of 119,716. If there is a remainder when the last period of figure has been moved down, place a decimal point after the figures already obtained in the root, annex two ciphers (00) to the remainder, and proceed as before until a sufficient number of decimals have been obtained.

380. Extracting the Cube Root.—Assume that the cube root of 80,621,568 is to be found. Beginning at the unit figure or decimal point, point off the number into periods of three figures each.

$$\begin{array}{rl} & 80'621'568\ \underline{|432} \\ 4 \times 4 \times 4 = & \underline{64} \\ 4^2 \times 300 = 4{,}800 & 16621 \\ 4^2 \times 300 \times 3 + 4 \times 30 \times 3^2 + 3^3 = & \underline{15507} \\ 43^2 \times 300 = 554{,}700 & 1114568 \\ 43^2 \times 300 \times 2 + 43 \times 30 \times 2^2 + 2^3 = & \underline{1114568} \end{array}$$

Find the greatest whole number, the cube of which does not exceed the value of the figures in the left-hand period (80), and write this number as the first figure in the root; 4 is the greatest whole number the cube of which does not exceed 80; subtract the cube of 4 from the left-hand period and move down the next period of three figures, and annex it to the remainder.

Multiply the square of the figure in the root by the constant 300; $4^2 \times 300 = 4800$), and find how many times this product is contained in the number 16,621. This gives a trial figure for the second figure of the root.

Now subtract from 16,621 the *sum* of the following products:

1. The square of the figure or figures already obtained in the root,

except the last one, multiplied by 300 and this product multiplied by the figure just obtained in the root: $4^2 \times 300 \times 3 = 16 \times 300 \times 3 = 14{,}400$.

2. The figure or figures already obtained in the root, except the last one, multiplied by 30, and this product multiplied by the square of the last figure obtained: $4 \times 30 \times 3^2 = 4 \times 30 \times 9 = 1{,}080$.

3. The cube of the last figure obtained: $3^3 = 3 \times 3 \times 3 = 27$.

If the sum of these various products is larger than 16,621, it indicates that the trial figure is too large, and a figure one unit smaller should be used. After having subtracted as directed, move down the next period of three figures, and annex it to the remainder, and proceed as before.

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Q925 Booth, F.
B72 Radiant energy and the
1921 ophthalmic lens. 52572

NAME	DATE DUE
E. Shaull	MAR 23 1931
Herbert L. Shapiro	DEC 10 1933 JAN 5 1934
	OCT 19 1940 MAY 19 1941
Yvonne Champreux	